

**Final Report on the  
International Comparison of Luminous Responsivity  
CCPR-K3.b**

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***Abstract***

During its 13<sup>th</sup> meeting in 1994 the Consultative Committee for Photometry and Radiometry (CCPR) decided to carry out a comparison of luminous responsivity to be organized by the BIPM. This comparison was classified as a key comparison by the 14<sup>th</sup> CCPR in 1997 and subsequently named CCPR-K3.b.

Fifteen laboratories took part in this comparison by sending calibrated photometers to the BIPM. The photometers were compared with a common set of reference photometers to deduce the relative differences between the calibrations made in the participating laboratories.

In this report the details of the procedure and the results are presented.

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## 1. Introduction

During the 13<sup>th</sup> meeting of the CCPR in 1994 it was decided to carry out a comparison of luminous responsivity using  $V(\lambda)$ -corrected detectors. This was the first time that the CCPR decided to carry out an international comparison of a photometric quantity using only detectors as transfer standards. So far in the history of the CCPR only lamps were used to compare photometric units, except for a mixed lamp / detector comparison in 1978 which was considered a trial comparison to assess the performance of lamps and detectors as transfer standards rather than a comparison of the units themselves [1]. The purpose of the current comparison is not to identify a 'best' photometer, but to compare luminous responsivity values determined by the laboratories. The present exercise was planned to give similar information about the coherence of national luminous intensity units in parallel with another comparison (key comparison CCPR-K3.a, [2]), prepared at the same time, using lamps as transfer devices. It was hoped that the outcome of this comparison would show whether state-of-the-art photometers could provide a quality of transfer standard for photometric units which would equal, or even surpass, that of lamps.

A working group consisting of CSIRO, NPL, OMH, PTB with the BIPM as the convenor was set up and recommended the parameters for the comparison.

## 2. The participants

The following table lists the participating laboratories:

<i>acronym</i>	<i>laboratory name</i>	<i>city</i>	<i>country</i>
BIPM	Bureau International des Poids et Mesures	Paris	
BNM-INM	Bureau National de Métrologie – Institut National de Métrologie	Paris	France
CSIC / IFA	Consejo Superior de Investigaciones Científicas / Instituto de Física Aplicada	Madrid	Spain
CSIRO	Commonwealth Scientific and Industrial Research Organization	Lindfield	Australia
HUT	Helsinki University of Technology	Espoo	Finland
IRL <sup>1</sup>	Industrial Research Limited	Lower Hutt	New Zealand
KRISS	Korean Research Institute of Standards and Science	Teajon	Rep. of Korea
NIM	National Institute of Metrology	Beijing	China
NIST	National Institute of Standards and Technology	Gaithersburg	USA
NPL	National Physical Laboratory	Teddington	U.K.
NRC	National Research Council	Ottawa	Canada
OFMET <sup>2</sup>	Eidgenössisches Amt für Messwesen	Wabern	Switzerland
OMH	Országos Mérésügyi Hivatal	Budapest	Hungary
PTB	Physikalisch-Technische Bundesanstalt	Braunschweig	Germany
SMU	Slovenský Metrologický Ústav	Bratislava	Slowakia
VNIIOFI	All Russian Research Institute for Optophysical Measurements	Moscow	Russia

**Table 1:** Participating laboratories.

<sup>1</sup> IRL : now MSL, Measurement Standards Laboratory

<sup>2</sup> OFMET : now METAS, Bundesamt für Metrologie und Akkreditierung / Swiss Federal Office of Metrology and Accreditation

### **3. The photometers**

The photometers used were commercially available ones. It was decided that they should be fully filtered and thermally stabilized. After contacting the different manufacturers three photometers were chosen, one from each of the following manufacturers: LMT, PRC Krochmann and Inphora, for the rest of this document they will be referred to as LMT, PRC and IPR, respectively.

The BIPM purchased one photometer of each type prior to the start of the comparison, henceforth referred to as BIPM-1, BIPM-2 and BIPM-3, serving as a reference group (Table 2). In a later stage four more photometers were purchased (BIPM-4 to BIPM-7). These photometers were modified to derive the BIPM detector-based luminous responsivity scale. BIPM-6 failed during the comparison.

<i>Photometer</i>	<i>Manufacturer</i>	<i>Serial Number</i>
BIPM-1	PRC	951016
BIPM-2	LMT	109508
BIPM-3	IPR	PO100
BIPM-4	LMT (modified)	796401
BIPM-5	LMT (modified)	796402
BIPM-6	PRC (modified)	970224-1
BIPM-7	PRC (modified)	970224-2

**Table 2:** List of photometers used at the BIPM. Photometers BIPM-1, BIPM-2 and BIPM-3 formed the reference group. BIPM-4 to BIPM-7 were used as transfer standards to represent the BIPM luminous responsivity units.

#### 4. Protocol of the comparison

Each participant was asked to send two photometers to the BIPM, together with their calibration report including the uncertainty budget and experimental conditions. The following table lists the photometers which were sent to the BIPM. The photometers are identified by the acronym of the laboratory and a running number. The table also gives the luminous responsivity<sup>3</sup> of each photometer as given by the participants before and after the devices were at the BIPM. Three laboratories opted to send three photometers. The BIPM participated with its four photometers BIPM-4, BIPM-5, BIPM-6 and BIPM-7 (Table 2).

<i>Photometer</i>	<i>Manufacturer</i>	<i>Serial Number</i>	<i>S (before) / (nA/lx)</i>	<i>S (after) / (nA/lx)</i>
BNM / INM-1	LMT	29645	48.25	48.27
BNM / INM-2	PRC	960221	11.69	
CSIC-1	IPR	130TH1	15.36	15.34
CSIC-2	IPR	PO143	17.08	17.04
CSIRO-1	LMT	39638	50.30	50.31
CSIRO-2	PRC	960321	11.82	11.82
CSIRO-3	IPR	0696PO112	16.41	16.43
HUT-1	PRC	960319-1F	11.73	11.71
HUT-2	PRC	960319-2F	12.15	12.14
IRL-1	PRC	9603	12.05	12.05
IRL-2	PRC	9605	11.55	11.54
KRISS-1	LMT	496212	53.57	53.53
KRISS-2	LMT	496213	53.09	53.04
KRISS-3	LMT	496215	53.26	53.23
NIM-1	PRC	961129	11.82	11.83
NIM-2	IPR	130 TH2	13.45	13.43
NIST-1	LMT	796421	51.48	51.42
NIST-2	LMT	796422	51.38	51.30
NPL-1	LMT	79605	51.28	51.26
NPL-2	PRC	960728	11.84	11.85
NRC-1	PRC	9602	12.06	12.04
NRC-2	PRC	9607	11.97	11.96
OFMET-1	LMT	296651	52.71	52.61
OFMET-2	LMT	296652	52.62	52.53
OFMET-3	LMT	296653	52.45	52.34
OMH-1	PRC	931223	11.90	11.89
OMH-2	PRC	950735	11.60	11.61
PTB-1	LMT	3966291	50.22	50.17
PTB-2	LMT	3966292	50.15	50.10
SMU-1	LMT	496301	52.25	51.71
SMU-2	LMT	496302	51.73	51.49
VNIOFI-1	LMT	896031	51.90	51.82
VNIOFI-2	LMT	896032	51.32	51.32

**Table 3:** List of photometers used in the comparison. Note: INM-2 failed shortly after arrival at the BIPM. The luminous responsivity values  $S(\text{before})$  and  $S(\text{after})$  are those the participants stated.

<sup>3</sup> The luminous responsivity is defined as the ratio of the photocurrent and the illuminance in the plane of the entrance aperture. Its unit is A / lx.

All photometers were generally measured against the reference group for the first time at the BIPM only a few days after their arrival. They were all measured at least on five different days. The devices were then returned to the originating laboratories where the participants checked them for drift in responsivity. They then reported the final luminous responsivity values to the BIPM.

Measurements were originally scheduled to be made at illuminant A and at an illuminance level of approximately 50 lx or lower. Alternatively a proposal had been made to perform measurements also with an illuminance of 500 lx or higher and at a distribution temperature of 3000 K. Only a few laboratories did these supplementary measurements. When the meeting of the 14<sup>th</sup> CCPR was advanced in date, it was decided to use the occasion to return the devices to the laboratories and because of the lack of time only measurements at illuminant A at about 40 lx were made at the BIPM.

All but one photometer showed sufficient stability during the time they were kept at the BIPM; the photometer INM-2 failed some time after arrival at the BIPM. After the first two series of measurements giving the same value it suddenly changed its responsivity twice by several percent. Visual inspection showed a fringe pattern, presumably Newton's rings, in the filter. The supposition is that the cement between the different layers in the filter failed and that the filter subsequently delaminated. Similar behavior in this type of photometers has been reported in two other cases.

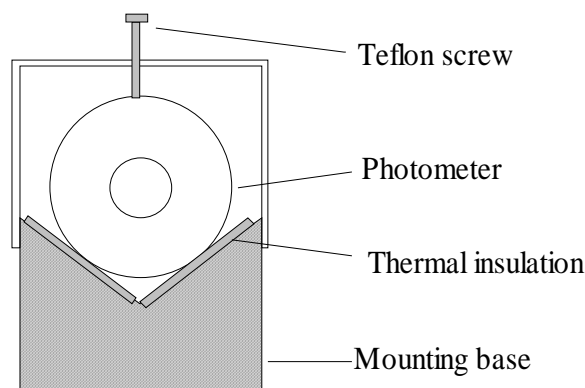
## 5. Preparatory measurements

The spectral responsivities of the photometers BIPM-1, BIPM-2 and BIPM-3 were measured using an experimental arrangement with a double monochromator, described elsewhere [3, 4]. The photometric bench employed for the comparison of the photometers is described later.

### 5.1 Photometer mounting

One observation made at the BIPM during these preparatory measurements was that the temperature stabilization of the photometer heads may not work correctly if the devices are clamped on a metal block with too large a mass. A small layer of thermal insulation (e.g. Teflon) easily avoided this problem.

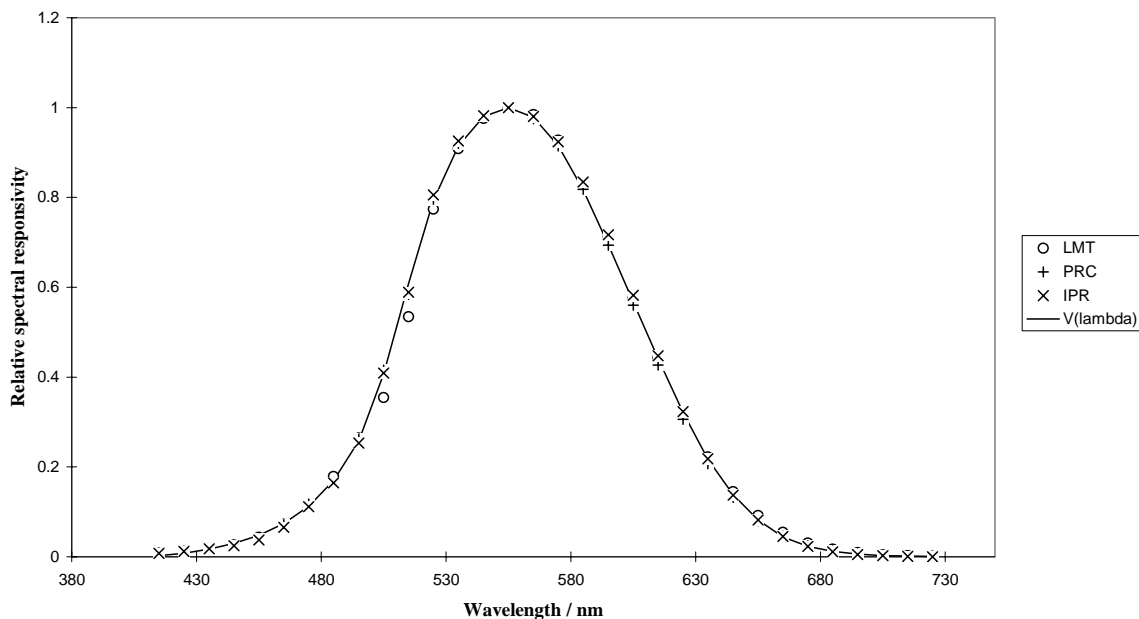
Figure 1 shows the mounting used at the BIPM with the insulation indicated. This information was given to the participants prior to the start of the comparison.



**Figure 1:** Thermal insulation of the photometer mount.

## 5.2 Spectral responsivity

The spectral responsivity curve was measured using a double monochromator arrangement described elsewhere [3, 4]. The three BIPM photometers forming the reference group and a reflection trap detector which had been calibrated against the BIPM cryogenic radiometer, were irradiated by a monochromator and compared with each other. The measurements were done in the following sequence: photometers A-B-C-C-B-A, taking the average of the two results for each photometer corrected for the small drift in the source radiance during a run. The relative spectral responsivity, normalized to unity at 555 nm is shown in Figure 2, compared with the  $V(\lambda)$  function.



**Figure 2:** Relative spectral responsivities of the three BIPM photometers forming the reference group compared against  $V(\lambda)$ .

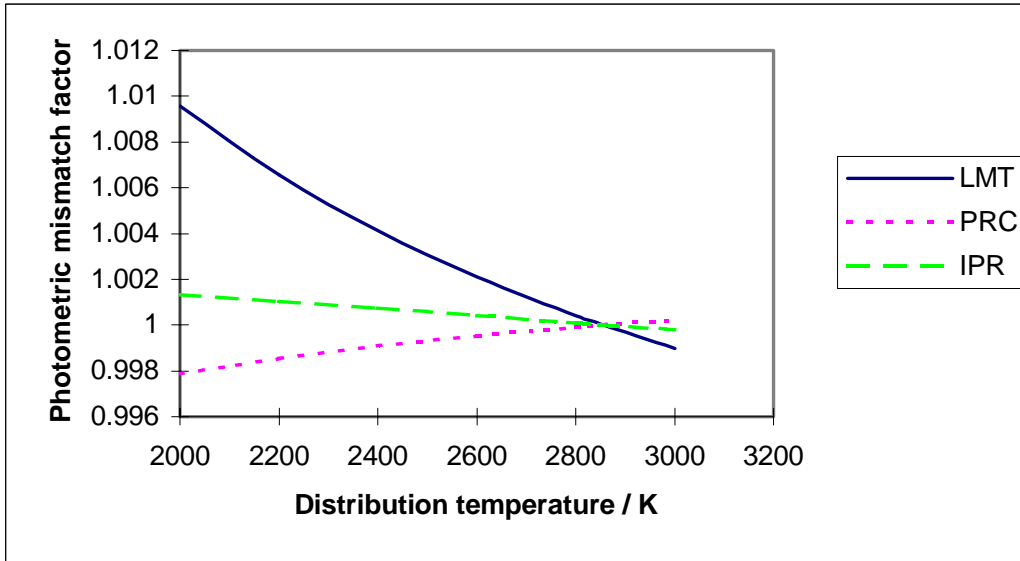
All three photometers have a fairly small  $f_1'$  value as defined in [5]:

$$f_1' \text{ (BIPM-1, PRC)} = 1.5 \%$$

$$f_1' \text{ (BIPM-2, LMT)} = 3.0 \%$$

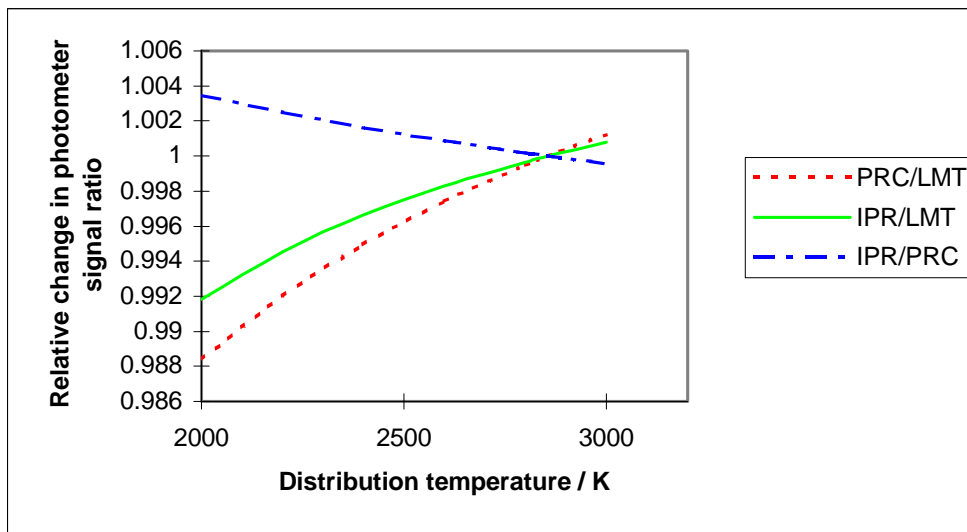
$$f_1' \text{ (BIPM-3, IPR)} = 1.5 \%$$

Partially filtered photometers with smaller  $f_1'$  values are available, but it was judged by the working group preparing the comparison that the uniformity of response over the entrance plane of the photometer was more important than a smaller  $f_1'$  value. The photometric mismatch as a function of the distribution temperature of a source  $F(T_d)$  [5] is shown in Figure 3a.



**Figure 3a:** Spectral mismatch for the BIPM photometers as a function of distribution temperature.

It is clear from the graph that a small error in the distribution temperature during the calibration will only have a negligible influence on the results. This is especially true when looking at the ratios between two photometers (Figure 3b).



**Figure 3b:** Change in the measured ratio of photometer luminous responsivity as a function of distribution temperature.

### 5.3 Influence of the last aperture in front of the photometers

Measurements were made to see if the diameter of the aperture closest to the photometer head could influence the results of the measurements. This was necessary because of the limited opening of the shutter. An iris diaphragm, effectively the one included in the shutter assembly, was placed about 20 cm from the photometers. The distance between the

photometers and the lamp was 2.5 m. The diameter of the aperture was adjusted such that its shadow line was just outside the largest entrance aperture of the photometers. The ratios between the photometer signals were taken and then the diaphragm was opened further so that new ratios could be measured. This series was continued until the largest possible opening of the diaphragm was reached. Figure 4 shows the result. The ratio of the signals LMT/PRC hardly changes as a function of the diameter of the diaphragm. The only small change visible occurs when the diameter corresponds closely to the opening of the photometer. This is probably due to diffraction. The ratio IPR/PRC however, alters dramatically with the beam diameter. The steepness of the curve changes twice before reaching a constant value. This behavior can be attributed to the shape of the front end of the IPR photometer head (see Figure 5). A series of concentric rings precedes the defining aperture on the  $V(\lambda)$  filter. Scattered light from the inside of these rings changes as the illuminated area increases. For the measurements on the photometric bench a diameter of 22 mm was chosen for the last diaphragm because the effect disappears at diameters greater than this value.

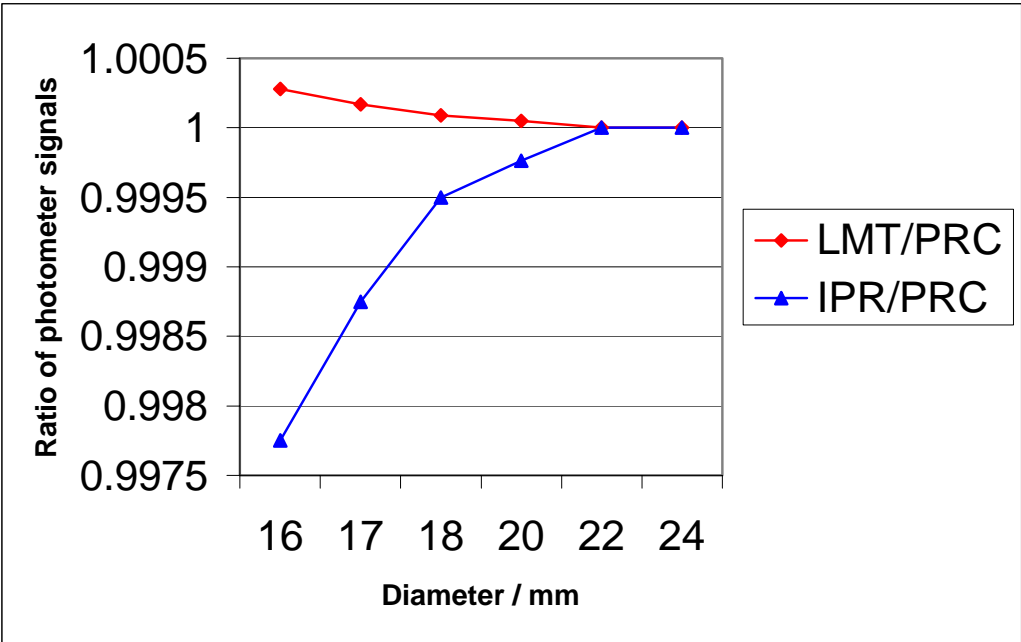


Figure 4: Influence of the aperture diameter on the photometer signal ratio.

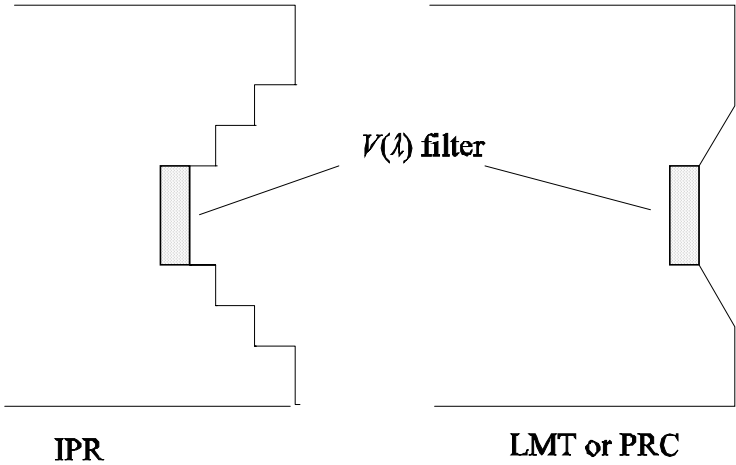
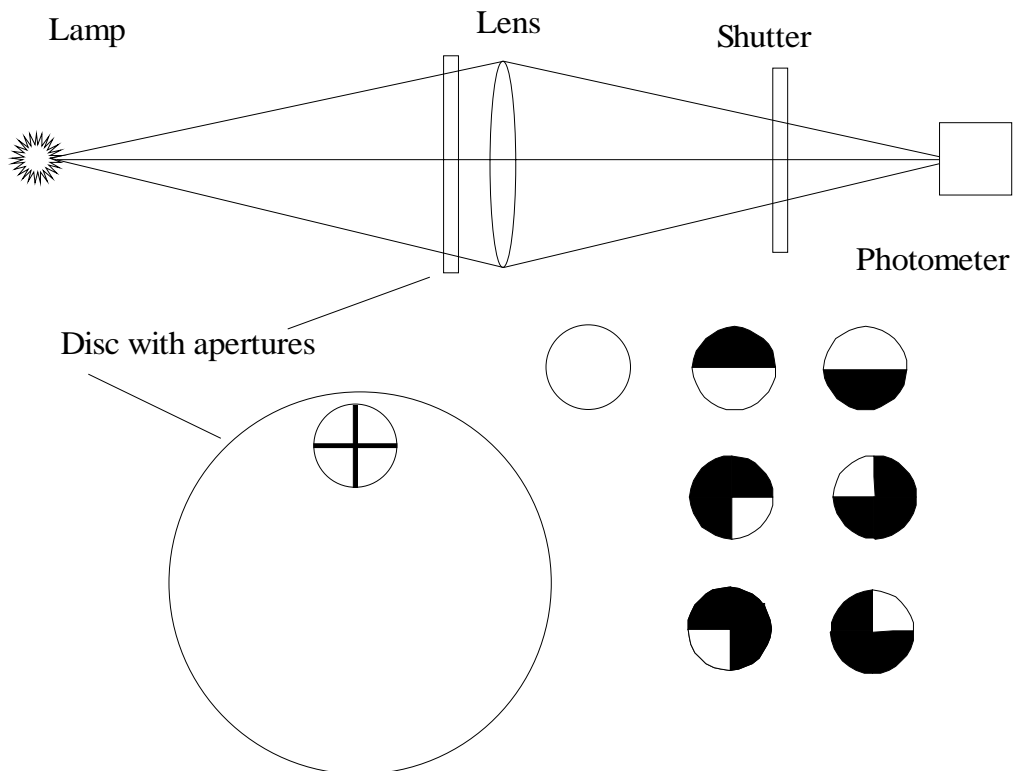


Figure 5: Shape of the front side of the photometers.

## 5.4 Linearity

All measurements at the BIPM were made at approximately the same illuminance level. Anticipating that some laboratories could use different parameters we checked the linearity of the photometers with the flux addition method. A lens formed the image of a lamp on to the photometer to be tested. Close to the plane of the lens an opaque disk containing a circular hole was inserted. The opening in the disk had two crossed bars (see Figure 6). Behind that disk, a second one could be rotated to insert holes having the forms shown in the figure, thus allowing a subdivision of the flux transmitted through the disk. Different combinations then allowed a check of the linearity.



**Figure 6:** Set-up for the linearity measurement. Explanation in the text.

Measurements were done with different sources and different focal lengths of the lens up to an illuminance level of about 2000 lx. No deviation from linearity was detected within the uncertainty of the measurements for all photometers. The measurements for the comparison at the BIPM were made at an illuminance level of about 40 lx.

## 5.5 Sensitivity to alignment

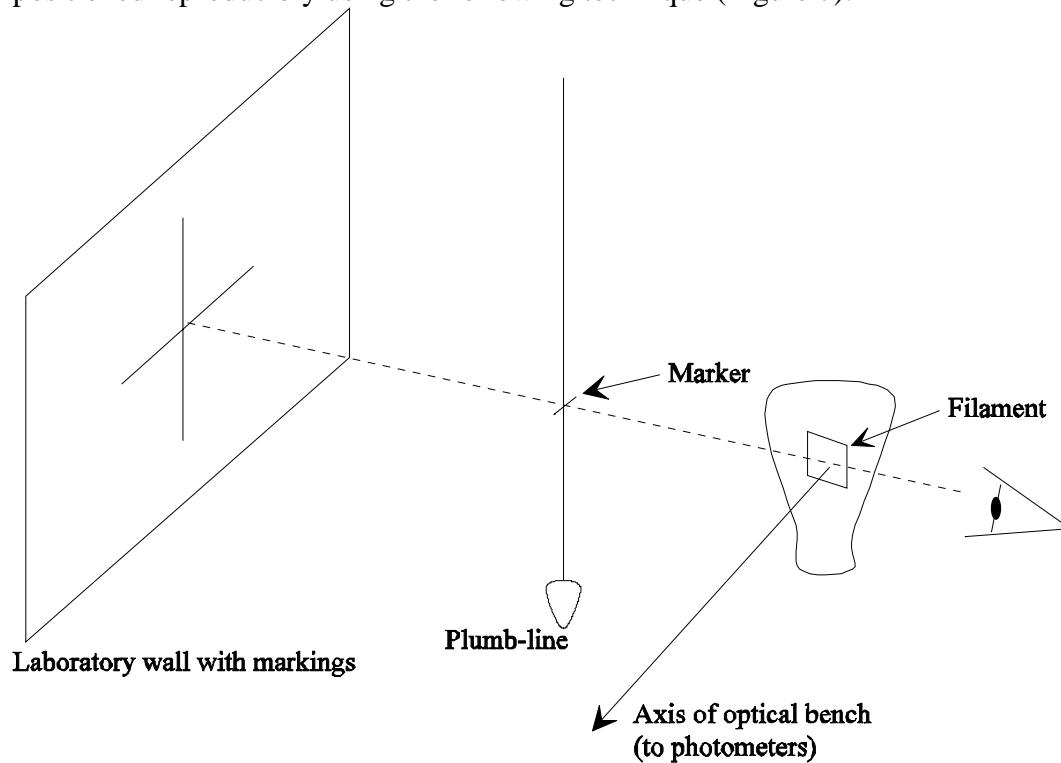
The influence of an inclination of the photometers with respect to the optical axis was studied by introducing such an inclination purposely and recording the difference found in the ratio between the different photometers, with and without inclination. It was found that with a source-detector distance of 2.5 m an inclination of about  $5^\circ$  resulted in a change of the measured ratio from 0.2 % to 0.4 %, depending on the photometer type. We estimated an uncertainty of  $0.3^\circ$  in the orientation of the photometers during our measurements resulting in a contribution of 0.03 % to the overall uncertainty.

## 6. Measurements for the Comparison

### 6.1 Alignment procedures

#### 6.1.1 Lamp position

The lamps used were Osram Wi 41 G type lamps, adjusted to a distribution temperature of  $2856 \text{ K} \pm 15 \text{ K}$ . According to the results shown in Figure 3b, the uncertainty in the distribution temperature has only a negligible influence on the results. The lamp was positioned reproducibly using the following technique (Figure 7):



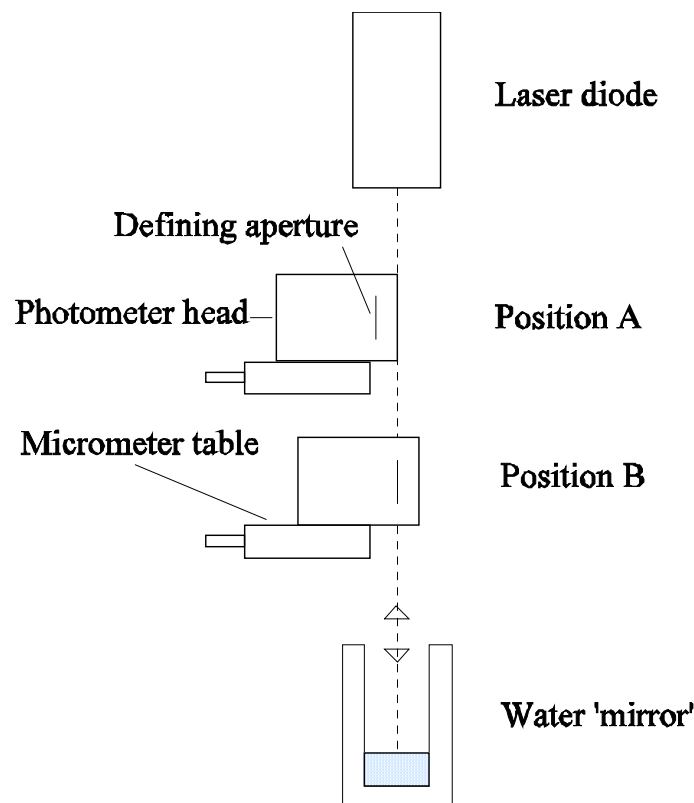
**Figure 7:** Method of lamp alignment.

The lamp filament was oriented vertically by visual comparison of the filament inclination with a plumb-line. The height of the filament was adjusted by altering its position until the middle of the filament coincided with a marker on the plumb-line. The lamp was then displaced along the axis of the photometric bench until the filament, the plumb-line and a vertical line on a wall in the laboratory, about 1.5 m away from the lamp, aligned. Finally the lamp was lit and the lamp holder rotated until the weak shadow of the filament became sharpest, indicating alignment of the filament perpendicular to the optical axis.

### 6.1.2 Distance

Three photometers were mounted at the same time on a translation stage which allowed displacement perpendicular to the optical axis<sup>4</sup>. This stage itself was mounted on a sliding table which could be manually displaced along the photometric bench in the direction of the optical axis.

The furthestmost part of the sliding-table was positioned at the point normally occupied by the lamp by placing it in line with the plumb-line and the vertical line on the wall, in a manner similar to that used for alignment of the lamp filament. A ruler on the photometric bench was then read using a microscope attached to the table. The table was then displaced by 2.5 m. A laser diode, adjusted to be vertical using the reflection of its beam from a water surface, was used to 'pinpoint' this position (Figure 8, position A). We estimate the uncertainty in the distance from this procedure to be 0.3 mm.



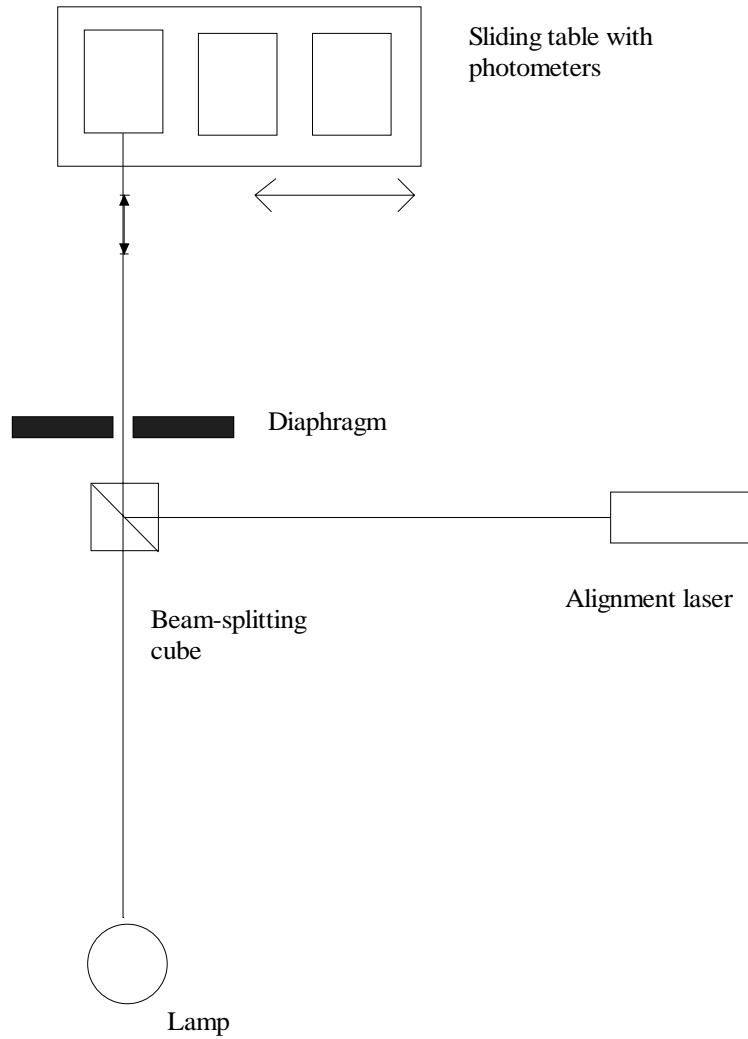
**Figure 8:** Alignment of the distance of the defining aperture.

The table was then moved again until one of the photometer front surfaces just touched the vertical laser beam indicating that the photometer surface was then 2.5 m from the lamp's filament. The final alignment for the limiting apertures was done relative to this position using small sliding tables fitted with micrometer screws (Figure 8, position B).

### 6.1.3 Photometer alignment

A beam splitting cube could be inserted in the optical path of the photometric bench. A laser beam reflected off the cube was then aligned parallel to the optical axis of the photometric bench (Figure 9).

<sup>4</sup> we define the optical axis as the line between the lamp and the photometer to be measured.



**Figure 9:** Photometer alignment.

The principal reflection from the cube determined the photometer position, while the secondary reflection from the side of the cube defined the lamp position. The cube was aligned once such that the beam hit the filament of the lamp, the latter having been mounted in the standard way to define the optical axis (paragraph 6.1.1).

The photometers were centered with respect to the beam and the first back reflection from the photometer head was used to align the heads perpendicular to the beam. The diaphragm closest to the photometers was the one in the shutter, which was opened to about 22 mm diameter and placed approximately 20 cm from the photometers. A series of baffles was used to minimize stray light. The choice of the diameter of the last diaphragm and its distance was made on the basis of the results of the preparatory measurements (paragraph 5.3).

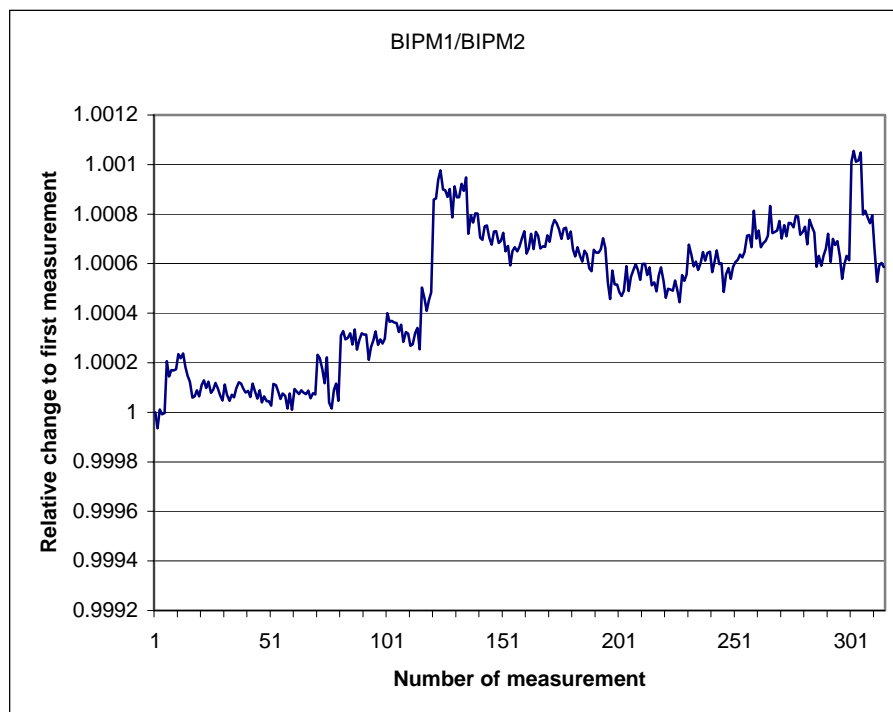
## 6.2 Comparison measurements

Three photometers could be mounted at the same time on the translation stage. Two of the three photometers in each run were from the BIPM reference group, the third one being one of the photometers from a participant. At least half an hour was allowed for thermal stabilization after the photometer had been connected to the power supply. The photometers were optically aligned using the technique described above.

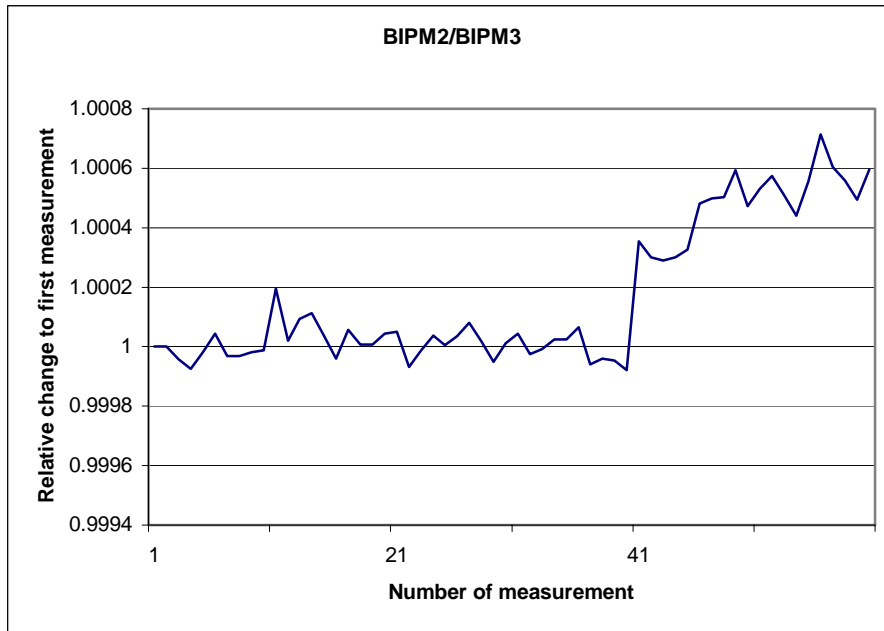
A measurement cycle consisted of five back and forth displacements of the translation stage, placing the different photometers in turn on the axis of the bench. Measurements were taken at each position, *back* and *forth*. This cycle was repeated five times before another photometer was mounted on the table. Each participant's photometer was measured at least five times on different days. Data were processed off-line.

## 7. Data analysis

As there were always two of the BIPM photometers on the bench together with one photometer from a participant, the ratio between the BIPM photometers can be used to check the stability of the reference group comprising the three BIPM photometers. Supplementary checks were done regularly by mounting all three BIPM photometers on the comparison bench and comparing them. Figures 10a and 10b show the ratios of the photometers of the reference group over four months.



**Figure 10a:** Stability of the reference photometers, normalized ratio BIPM1 / BIPM2



**Figure 10b:** Stability of the reference photometers, normalized ratio BIPM2 / BIPM3

Although some structure is present, the relative standard uncertainties of the ratios monitored are relatively small. This relatively small spread for filtered detectors is smaller than the uncertainties of the scales to be compared. The fact that the ratios of the responsivities of photometers of different type are stable indicates that the individual photometers are stable, as it can be assumed that different types would not exhibit similar fluctuations in their responsivity. The contribution to the relative standard uncertainty of the comparison was estimated as 0.08 % (see Table 5).

To derive the differences between the participants' luminous responsivity units, all LMT and IPR photometers were normalized with respect to BIPM-1 (PRC type). The PRC photometers were normalized with respect to BIPM-2 (LMT type) and the BIPM-2 / BIPM-1 ratio was used to calculate the results of the comparison. This procedure was adopted owing to the much larger size of the PRC photometer heads and because it avoided continual remounting of the mechanical parts on the photometric bench each time a measurement was made. The influence of the amplifier gains was studied by inverting the amplifier connections. It was found that the effect was within the repeatability of the results ( $< 2$  parts in  $10^4$ ), so that it was not taken into account in the calculation.

To compare the photometer calibrations the following calculation was made:

$$s_{\text{ref},i} = s_i y_{\text{ref}} / y_i$$

where  $s_{\text{ref},i}$  represents the calibration of the BIPM reference photometer by the  $i$ -th photometer of the comparison batch,  $s_i$  is the responsivity for that photometer as given by the participant and  $y_{\text{ref}}$ ,  $y_i$  are the signals measured from the reference photometer and the participant's photometer, respectively. In that way the BIPM reference photometer was calibrated with respect to each individual participating photometer.

## 8. Results

The CCPR working group on key comparisons decided how the key comparison reference value (KCRV) should be calculated and its decision was approved by the 15<sup>th</sup> CCPR. The key comparison reference value should be calculated as the weighted mean with the inverse square of the uncertainties as weight. It was considered that some of the uncertainties stated were possibly too small and that therefore a 'cut-off' value for the uncertainties should be used, which was fixed to be 0.2 %. The cut-off was only to be used in the calculation of the reference value but the originally given uncertainties by the laboratories should be used in the tables and graphs. Laboratories concerned by this rule are marked with an '\*' in Table 4 and in Figure 11. The KRISS declared that they had identified a problem with their reference photometer and the working group decided to exclude the KRISS from the calculation of the reference value.

For each laboratory the results obtained for the different photometers were first averaged, then with the exception of the KRISS result, the weighted mean value of these averages for all laboratories was calculated using the inverse square of the uncertainties as a weight after the application of a cut-off of 0.2 % as the minimum uncertainty. Both the 'before' and 'after' values were used for the calculation of this key comparison reference value, except for the BNM/INM value because of the failure of the second photometer during the measurements at the BIPM. The percentage difference of the mean of each laboratory from the key comparison reference value is shown in Figure 11 and in Table 4. Also indicated in the figure is the uncertainty of the reference value of 0.06 % which is calculated as the standard uncertainty  $u_0$  of the weighted mean using the formula

$$u_0 = \sqrt{\frac{1}{\sum \frac{1}{u_i^2}}}$$

where the  $u_i$  are the uncertainties stated by the participants after application of the cut-off.

In Figure 12 the percentage difference for each individual photometer before and after the BIPM measurements is plotted against the same reference value as in Figure 11. As some laboratories sent three photometers, their results would have entered with a greater weight than those with only two photometers if the weighted mean of the comparison had been taken from the individual photometer data as opposed to the mean per laboratory. For clarity in Figure 12 the uncertainties are indicated only for the 'before' values.

There have been several corrections of data during the draft stage. The corrections of clerical errors and of misinterpretations of some of the reported laboratory results by the pilot after Draft A are listed and explained in Appendix 1. There were requests from CSIC/IFA and OFMET to correct their data after Draft A due to technical reasons. These revised data were not accepted for the Final Report but are shown in Appendix 2 for information. Only minor corrections as below requested by HUT and NPL were accepted.

The HUT has observed a drift in one of its photometers over a period of more than one year. The data measured at the BIPM agree with this observed drift. Consequently the HUT asked to correct the 'before' and 'after' values for this photometer by  $-0.17$  % and  $+0.21$  %, respectively, resulting in a change of the HUT value by only 0.02 %.

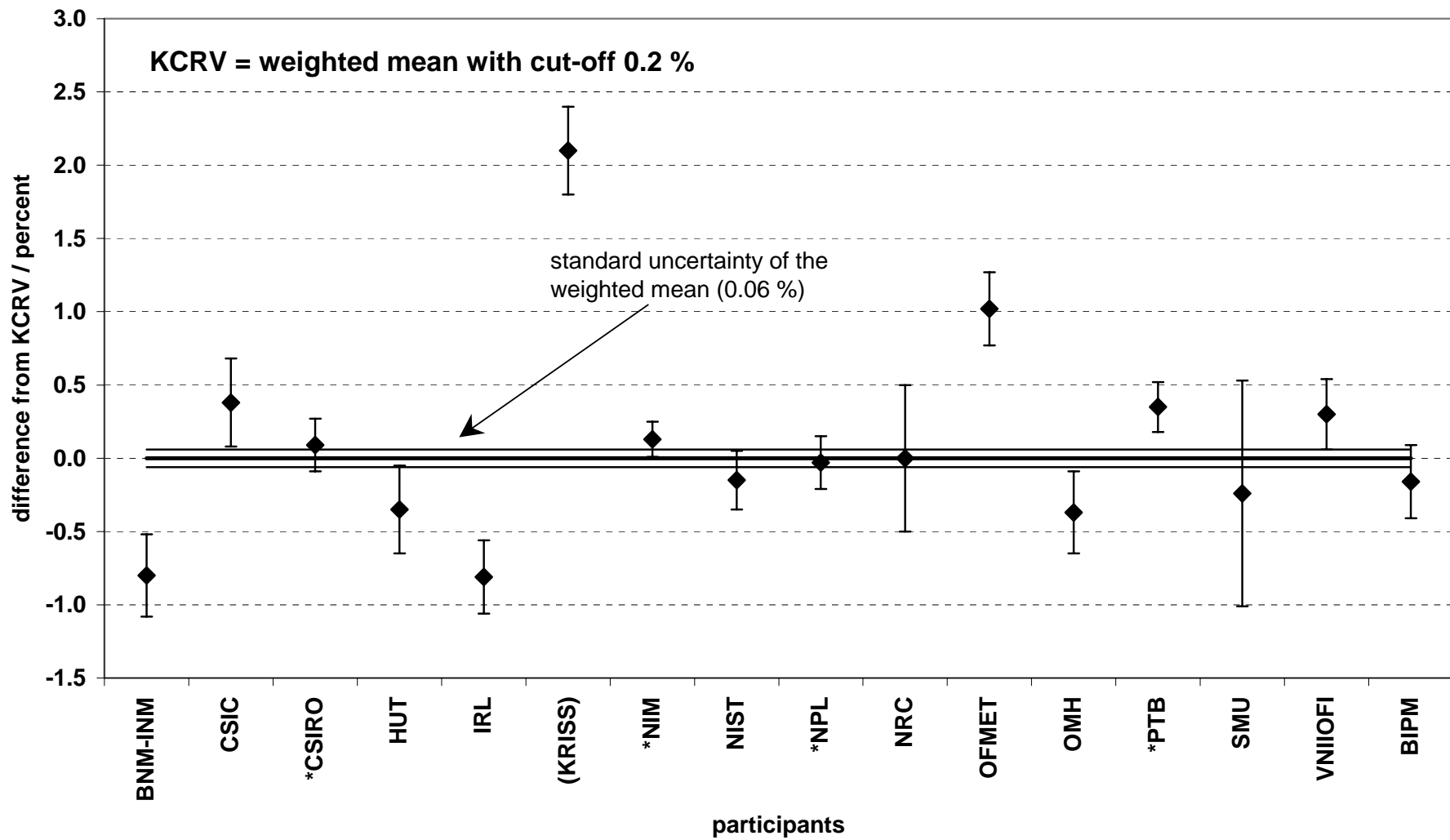
NPL has corrected an error it made in calculating the stated uncertainty of the mean value reported in Draft A. The standard uncertainty value was corrected from 0.25 % to 0.18 % in Draft B and the Final Report.

participant	percentage difference from key comparison reference value	uncertainty ( $k=1$ )
BNM-INM	-0.80	0.28
CSIC	0.38	0.30
CSIRO *	0.09	0.18
HUT	-0.35	0.30
IRL	-0.81	0.25
(KRISS)	2.10	0.30
NIM *	0.13	0.12
NIST	-0.15	0.20
NPL *	-0.03	0.18
NRC	0.00	0.50
OFMET	1.02	0.25
OMH	-0.37	0.28
PTB *	0.35	0.17
SMU	-0.24	0.77
VNIIOFI	0.30	0.24
BIPM	-0.16	0.25
BIPM (L85)	-0.26	0.50

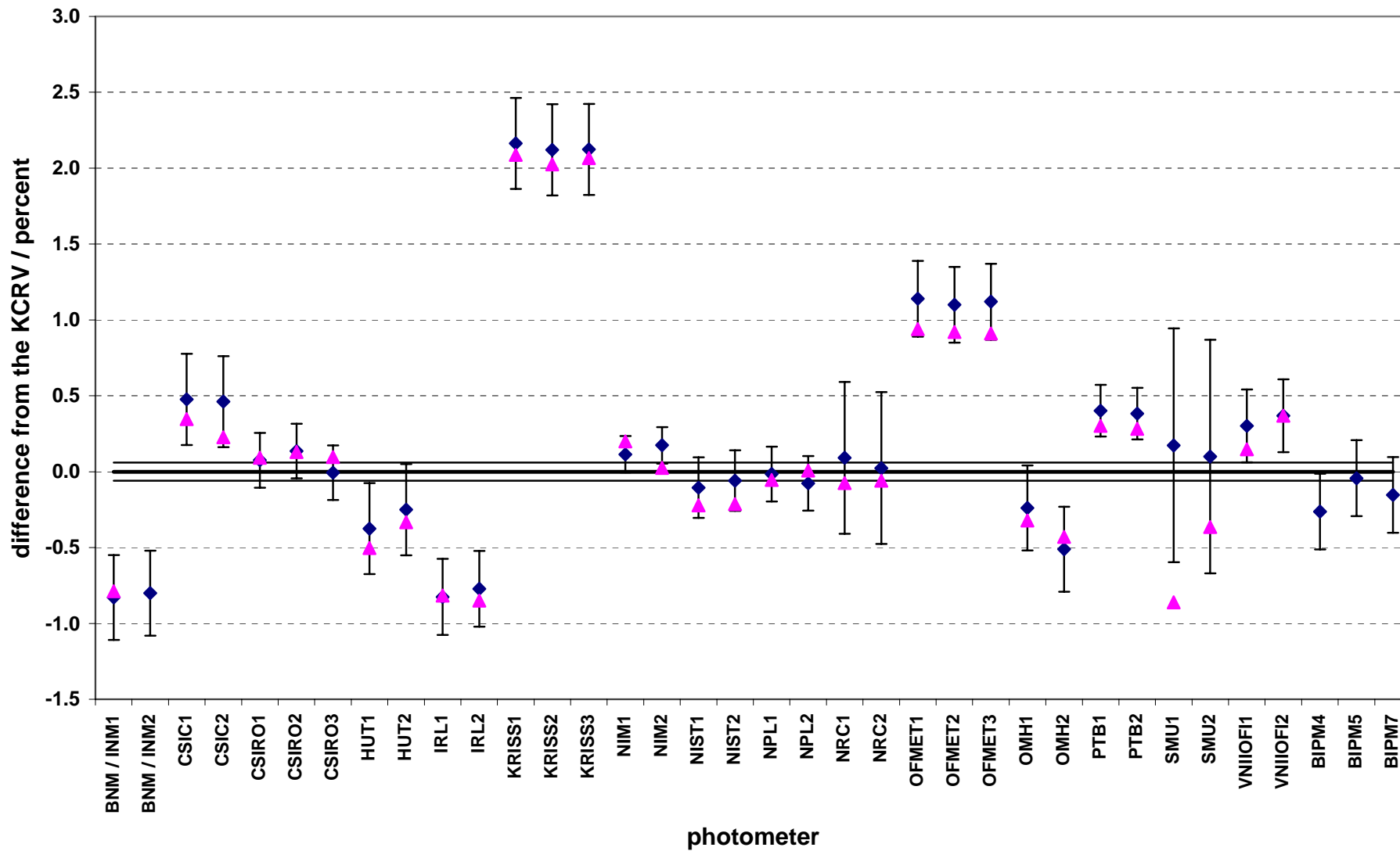
**Table 4:** Percentage difference from the key comparison reference value (weighted mean with cut-off of 0.2 %) for the different participants. The uncertainties are those given by the laboratories. The uncertainties of the laboratories marked with an \* are below the cut-off of 0.2 % and were replaced by 0.2 % in calculating their weights. The KRISS result was not used in the calculation of the weighted mean since its result was considered to be an outlier. The result BIPM (L85) was obtained by calibrating the BIPM photometers against the luminous intensity lamps maintaining the mean value of the 1985 comparison of lum. intensity.

It can be seen from Figure 11 that the overall spread is probably somewhat higher than could have been hoped for, but that most laboratories do agree well with the reference value. Twelve laboratories agree with the reference value within  $2\sigma$ . The reference value agrees well with the mean value from the 1985 international comparison of luminous intensity using lamps as it is maintained in the form of a group of lamps at the BIPM (see entry BIPM (L85) in Table 4).

The standard deviation of the participants' results (without KRISS) is 0.47 % and thus similar to the standard deviation observed in the comparison CCPR-K3.a made with lamps.



**Figure 11:** Results of the comparison averaged over all photometers per laboratory. The graph shows the relative difference from the weighted mean in percent for each laboratory. The uncertainties correspond to the standard uncertainties ( $k=1$ ) stated by the laboratories and do not include the uncertainty of the comparison itself.



**Figure 12:** Results of the comparison for each individual photometer. The graph shows the relative difference from the same reference value as in Figure 11 in percent for each photometer before (diamonds) and after (triangles) the measurements at the BIPM. The uncertainties correspond to the standard uncertainties ( $k=1$ ) stated by the laboratories and do not include the uncertainty of the comparison itself.

## 9. Uncertainties of the comparison

UNCERTAINTY CONTRIBUTION	RELATIVE STANDARD UNCERTAINTY $\times 100$
Measurement repeatability (A type)	0.02
Stability of reference group	0.08
Linearity	0.01
Alignment (pivot)	0.03
Alignment (distance)	0.02
<b>Total relative standard uncertainty</b>	<b>0.09</b>

**Table 5:** Uncertainty budget for the comparison of luminous responsivity.

The largest contribution to the uncertainty of the comparison is due to the stability of the reference group. The spectral mismatch of the reference and the distribution temperature contribute only a negligible factor to the uncertainties.

The total relative standard uncertainty of the comparison is 0.09 %, which is considerably smaller than the calibration uncertainties of the photometers.

## 10. Realization of the photometric quantities in the different laboratories

The following short descriptions about the realization of the photometric unit and the method of calibration are taken from the laboratory reports of the calibration.

### *BIPM:*

Measurement of the spectral responsivity against a trap detector calibrated at a cryogenic radiometer and separate determination of the aperture area. An additional calibration was made using the group of luminous intensity lamps maintaining the mean value of the 1985 comparison of luminous intensity. The result of this calibration is shown in the last line of table 4 as BIPM (L85).

### *BNM/INM:*

Comparison with a group of standard lamps at 2800 K. The value for these lamps was known from the 1983 radiometric realization of the candela using ESR's. The BNM-INM has informed us that they are currently completing a new realization of the unit candela. Preliminary results of that study seem to indicate that the difference between the old and the new realization is of the order of 0.8 %.

### *CSIC / IFA:*

Comparison with three standard photometers. The standard photometers were measured in relative spectral responsivity and then calibrated at Ar laser wavelengths against an Si detector, whose responsivity is traceable to a cryogenic radiometer.

### *CSIRO:*

Comparison with a room-temperature,  $V(\lambda)$ -corrected ESR.

*HUT:*

Comparison of the photometer with a reference photometer consisting of a characterized trap detector fitted with a  $V(\lambda)$  filter and a precision aperture. The trap responsivity is traceable to the HUT cryogenic radiometer. The photometers used were compared directly with the reference photometer.

*IRL:*

Comparison with standard photometers. Their responsivity was determined using a trap-detector spectral responsivity scale which had been derived from an internal quantum efficiency model and silicon photodiode reflectance measurements; cryogenic radiometer measurements were used for renormalization.

*KRISS:*

No information was communicated.

*NIM:*

Comparison with a group of standard lamps. No information was communicated on how the lamp values were determined.

*NIST:*

A group of characterized photometers traceable to the cryogenic radiometers form a reference group. The photometers were compared directly with this group.

*NPL:*

The measurement of responsivity of photometers at NPL is carried out by comparison with the NPL scale of luminous intensity. The derivation of the NPL scale of luminous intensity, is based on a radiometric realization of the candela using the NPL absolute cryogenic radiometer. The lamps used to disseminate the scale are calibrated against reference photometers which are calibrated against the NPL cryogenic radiometer. The realization of the candela at NPL is described in the paper: Metrologia 25, 29-40 (1988).

*NRC:*

Comparison with a group of reference lamps. The luminous intensity values of the lamps are obtained by comparison with the NRC 1985 realization of the candela using  $V(\lambda)$ -corrected ESR's and silicon cells.

*OFMET:*

Comparison with a group of lamps. The luminous intensity values of the lamps are known from the 1985 CCPR comparison adding the recommended correction of 1 %. These values were confirmed by provisional results of a radiometric realization.

After the OFMET had received draft A and after knowing the provisional results from the luminous intensity comparison they communicated revised values based on the latter comparison.

*OMH:*

The luminous intensity unit at OMH is realized and maintained by a group of three  $V(\lambda)$ -corrected standard photometers. The absolute spectral responsivity of the photometers is based on the predictable quantum efficiency method of the silicon photodiode and some characterized trap detectors.

*PTB:*

At the PTB the luminous intensity unit is realized annually, based on the cryogenic radiometers in the clean-room center.  $V(\lambda)$ -corrected photometers are used as transfer standards between the radiometers and the network of photometers and lamps for the realization and maintenance of the candela. The stability of the network is higher than the uncertainty of a realization, so it is used to average over several realizations. The PTB photometric units have remained unchanged since the CCPR comparison in 1985.

*SMU:*

Calibration of a spectrally characterized photometer. The absolute values were calibrated against a QED-200.

*VNIIOFI:*

Comparison with a standard lamp. No information was communicated on how the lamp value was determined.

## **11. Conclusions**

The first international comparison of a photometric quantity using only photometers was carried out. The photometers have generally proven to be sufficiently stable and robust for such a comparison. The agreement of the results of the different laboratories is fairly good, with most participants agreeing within the stated uncertainties. The spread of the results, expressed as standard deviation of the results, is comparable to the spread observed in a similar comparison made with lamps as transfer standards.

## **12. Acknowledgements**

The authors would like to thank P. Zabarel who did a major contribution to the setting up and initial testing of the facilities used for this comparison.

## **13. References**

- [1] Comité Consultatif de Photométrie et Radiométrie, 10<sup>th</sup> session, 1982, p75.
- [2] G. Sauter, D. Lindner, M. Lindemann, "CCPR Key Comparisons K3a of Luminous Intensity and K4 of Luminous Flux with Lamps as Transfer Standards", PTB-Report, PTB-Opt-62, Braunschweig, December 1999, ISBN 3-89701-471-8.
- [3] R. Köhler, R. Goebel, R. Pello, "Results of the international comparison of spectral responsivity of silicon photodetectors", Metrologia, 32, 1996, 463-468.
- [4] R. Köhler, R. Goebel, R. Pello, "Report on the international comparison of spectral responsivity of silicon photodetectors", Report BIPM-94/9.
- [5] CIE publication 53 / 1982.

## **Appendix 1**

### **Corrections of clerical errors in Draft A and Draft B**

#### **1. BNM/INM**

Following their "after" measurement, BNM/INM revised their "before" values and reported the revised "before" and "after" values. The laboratory also requested to use only "before" data for one of the photometers due to a significant change detected. Draft A, however, did not make these corrections by mistake of the pilot laboratory, and used only the original "before" values. This error was corrected in Draft B and the Final Report.

#### **2. NPL**

Following its "after" measurement, NPL revised their "before" value and reported the mean of the "after" and the revised "before" values to BIPM. The change of the "before" measurement was due to a drift of the working standard lamps and was on average 0.26 %. The mean values were erroneously interpreted by the pilot as results of the "after" measurement in Draft A. This error was corrected in Draft B and the Final Report.

#### **3. NRC**

The results given in this Final Report for the two NRC photometers are different from those shown in the previous drafts. This is due to a correction of a misunderstanding of the communicated data by the pilot. The error was already discovered after Draft A but forgotten to be taken into account in Draft B. The effect of this change on NRC's data is 0.08 %.

## Appendix 2

### Revised results submitted after Draft A

#### 1. CSIC/IFA

The CSIC/IFA observed a too large infra-red response of their photometers. This fact was stated in the original calibration report. The CSIC later decided to apply a correction of  $-0.14\%$  to their calibration values to subtract the infra-red contribution from the luminous responsivity. In the Draft B of this report this correction had been applied. In the final data reduction we removed this correction because the IR response contributes to the measured photo-current and thus must be included in the luminous responsivity value assigned to the photometer.

#### 2. METAS/OFMET

The results of METAS/OFMET shown in this report are based on a group of luminous intensity lamps which maintained the international mean value of the 1985 luminous intensity comparison.

At the time of the luminous responsivity comparison CCPR-K3.b METAS was working on a new detector-based realization. This realization was not yet available at the beginning of the comparison. However, after the circulation of the Draft A report in January 1998 the new realization was completed and used to calculate improved luminous responsivity values for the three photometers. The uncertainty remained unchanged ( $0.25\%$ ). On average, the new values were  $0.52\%$  lower than the original ones. These modified data were originally accepted by the other participants and used in the subsequent draft reports. The Working Group on Key Comparisons of the CCPR discussed this subject again in its meeting on 15 June 2003 and decided that this data modification - having occurred after the distribution of the Draft A - was against the principles of the *Guidelines for CIPM key comparisons*. The official result given in this (second) Final Report is therefore based on METAS' original results.

For information this appendix shows the results based on the new detector-based luminous responsivity realization of METAS. The following table shows the luminous responsivities of the three photometers based on the old and the new realization.

photometer	old realization		new realization		
	"before"	"after"	"before"	"after"	
OFMET-1	52.71	52.61	52.36	52.39	nA/lux
OFMET-2	52.62	52.53	52.34	52.31	nA/lux
OFMET-3	52.45	52.34	52.09	52.13	nA/lux

Table A2.1: Submitted change of the METAS/OFMET results.

The results of all participants, calculated using the new METAS results are shown in the table below.

participant	percentage difference from key comparison reference value	uncertainty ( $k=1$ )
BNM-INM	-0.77	0.28
CSIC	0.41	0.30
CSIRO *	0.12	0.18
HUT	-0.32	0.30
IRL	-0.78	0.25
(KRISS)	2.14	0.30
NIM *	0.16	0.12
NIST	-0.11	0.20
NPL *	0.00	0.18
NRC	0.03	0.50
OFMET	0.53	0.25
OMH	-0.34	0.28
PTB *	0.38	0.17
SMU	-0.20	0.77
VNIIOFI	0.33	0.24
BIPM	-0.12	0.25
BIPM (L85)	-0.23	0.50

**Table A2.2:** Recalculation of the K3.b results using the revised METAS/OFMET values. For further details see the caption of Table 4 of the main report.

Compared to Table 4 of the Final Report, all differences from the KCRV (expressed in percent) are increased by 0.03 %, being the difference between the KCRVs calculated with the original and the new METAS values. The METAS result improves from 1.02 % to 0.53 %.

## Appendix 3

### Degrees of Equivalence for CCPR-K3.b

M. Stock, BIPM

In the main text of this Final Report the results are presented in the following form (Table 4):

participant	100 x relative deviation from key comparison reference value	100 x relative uncertainty ( $k=1$ )
BNM-INM	-0.80	0.28
CSIC	0.38	0.30
CSIRO *	0.09	0.18
HUT	-0.35	0.30
IRL	-0.81	0.25
(KRISS)	2.10	0.30
NIM *	0.13	0.12
NIST	-0.15	0.20
NPL *	-0.03	0.18
NRC	0.00	0.50
OFMET	1.02	0.25
OMH	-0.37	0.28
PTB *	0.35	0.17
SMU	-0.24	0.77
VNIIOFI	0.30	0.24
BIPM	-0.16	0.25

**Table A3.1:** Relative difference from the key comparison reference value (weighted mean with cut-off for the relative uncertainty of  $2 \times 10^{-3}$ ) for all participants. The uncertainties are the calibration uncertainties given by the laboratories. The relative uncertainties of the laboratories marked with an \* are below the cut-off and were replaced by  $2 \times 10^{-3}$  in calculating their weights. The KRISS result was not used in the calculation of the weighted mean since its result was considered to be an outlier.

The reference value is calculated as the weighted mean using a cut-off of  $2 \times 10^{-3}$ . The uncertainties given in Table A3.1 are those claimed by the participants *for their calibration*. To state the degrees of equivalence as defined in the MRA, the *uncertainty of the deviation from the reference value* is needed, which includes the calibration uncertainty, the uncertainty of the comparison at the BIPM, the uncertainty of the reference value and the correlation between the participants' uncertainties and that of the reference value.

Due to the application of an uncertainty cut-off in the calculation of the weighted mean  $x_w$ , the uncertainty of the deviation  $d_i$  cannot be calculated simply by using the formula which is valid for the uncertainty of the deviation from the (normal) weighted mean:

$$u^2(d_i) = u_i^2 - u^2(x_w).$$

As shown in Appendix 4, the more complicated formulas

$$u^2(d_i) = u_i^2 + u^2(x_w) - \frac{2 \frac{u_i^2}{v_i^2}}{\sum_j \frac{1}{v_j^2}} \quad \text{and} \quad u^2(x_w) = \frac{\sum_i \frac{u_i^2}{v_i^4}}{\left( \sum_j \frac{1}{v_j^2} \right)^2}$$

have to be used, where the  $v_i$  are adjusted uncertainties defined by  $v_i = \max(u_i, u_{\min})$  with  $u_{\min}$  being the cut-off value. The uncertainty of the reference value thus calculated is  $u(x_w) = 6.0 \times 10^{-4}$ .

To include the relative uncertainty of the comparison at the BIPM of  $9 \times 10^{-4}$  (Table 5), it will be added quadratically to the uncertainty of the deviation defined above. It would probably have been better to include this contribution *before* the calculation of the weights needed for the determination of the reference value, but the choice was made earlier during the preparation of the Final Report not to do so. The uncertainties obtained by this calculation are shown in Table A3.2.

participant	100 x relative deviation from key comparison reference value	100 x relative uncertainty ( $k=1$ ) of deviation	100 x relative uncertainty ( $k=2$ ) of deviation
BNM-INM	-0.80	0.29	0.57
CSIC	0.38	0.31	0.61
CSIRO *	0.09	0.19	0.39
HUT	-0.35	0.31	0.61
IRL	-0.81	0.26	0.51
(KRISS)	2.10	0.32	0.64
NIM *	0.13	0.15	0.30
NIST	-0.15	0.21	0.42
NPL *	-0.03	0.19	0.39
NRC	0.00	0.50	1.01
OFMET	1.02	0.26	0.51
OMH	-0.37	0.29	0.57
PTB *	0.35	0.19	0.37
SMU	-0.24	0.77	1.54
VNIOFI	0.30	0.25	0.49
BIPM	-0.16	0.26	0.51

**Table A3.2:** Degrees of equivalence for CCPR-K3.b. The uncertainty includes the calibration uncertainty of the participants, the uncertainty of the comparison at the BIPM and that of the reference value, taking into account its correlation with the participants' uncertainties (except for KRISS, whose result did not contribute to the reference value).

The bilateral differences are calculated from the deviations of the two laboratories from the reference value:

$$d_{i,j} = d_i - d_j = (x_i - x_w) - (x_j - x_w) = x_i - x_j$$

They do not depend on the reference value. The uncertainties of the bilateral differences are calculated as the quadratic sum of the calibration uncertainties plus two times the uncertainty of the comparison at the BIPM. Since by far the largest contribution to the latter is the stability of the reference group, these contributions are assumed to be uncorrelated. The uncertainty of the reference value does not contribute to the uncertainty of the bilateral differences. The matrix of bilateral differences and the associated uncertainties is shown on the next page.

Key comparison CCPR-K3.b

MEASURAND: Luminous responsivity of photometers

The individual measurements,  $x_i$ , of the participating laboratories correspond to the results of a virtual calibration of the luminous responsivity of the same photometer.

Key comparison reference value: the KCRV is calculated as the weighted mean of the participants results, weighted by the inverse square of the uncertainties with the application of a minimum uncertainty cutoff of 0.2 %. The KRIS is excluded from the calculation of the KCRV

The degree of equivalence of each laboratory with respect to the reference value is given by a pair of numbers:

$D_i = (x_i - x_{KCRV})$  and  $U_i$ , its expanded uncertainty ( $k = 2$ ), both expressed in relative units  
 $U_i^2 = 2^2(u_i^2 + u_{KCRV}^2 - u_{corr}^2 + u_{transfer}^2)$ , where  $u_{corr}$  is the correlation between  $u_i$  and  $u_{KCRV}$

The degree of equivalence between two laboratories is given by a pair of numbers:

$D_{ij} = D_i - D_j = (x_i - x_{KCRV}) - (x_j - x_{KCRV})$  and  $U_{ij}$ , its expanded uncertainty ( $k = 2$ ), both expressed in relative units  
 $U_{ij}^2 = 2^2(u_i^2 + u_j^2 + 2u_{transfer}^2)$

Lab j →

Lab i ↓			BNM-INM		CSIC		*CSIRO		HUT		IRL		(KRIS)		*NIM		NIST		*NPL		NRC		OFMET		OMH		*PTB		SMU		VNIOFI		BIPM	
	$D_i$ / 10 <sup>-2</sup>	$U_i$ / 10 <sup>-2</sup>	$D_{ij}$ / 10 <sup>-2</sup>	$U_{ij}$ / 10 <sup>-2</sup>	$D_{ij}$ / 10 <sup>-2</sup>	$U_{ij}$ / 10 <sup>-2</sup>	$D_{ij}$ / 10 <sup>-2</sup>	$U_{ij}$ / 10 <sup>-2</sup>	$D_{ij}$ / 10 <sup>-2</sup>	$U_{ij}$ / 10 <sup>-2</sup>	$D_{ij}$ / 10 <sup>-2</sup>	$U_{ij}$ / 10 <sup>-2</sup>	$D_{ij}$ / 10 <sup>-2</sup>	$U_{ij}$ / 10 <sup>-2</sup>	$D_{ij}$ / 10 <sup>-2</sup>	$U_{ij}$ / 10 <sup>-2</sup>	$D_{ij}$ / 10 <sup>-2</sup>	$U_{ij}$ / 10 <sup>-2</sup>	$D_{ij}$ / 10 <sup>-2</sup>	$U_{ij}$ / 10 <sup>-2</sup>	$D_{ij}$ / 10 <sup>-2</sup>	$U_{ij}$ / 10 <sup>-2</sup>	$D_{ij}$ / 10 <sup>-2</sup>	$U_{ij}$ / 10 <sup>-2</sup>	$D_{ij}$ / 10 <sup>-2</sup>	$U_{ij}$ / 10 <sup>-2</sup>	$D_{ij}$ / 10 <sup>-2</sup>	$U_{ij}$ / 10 <sup>-2</sup>	$D_{ij}$ / 10 <sup>-2</sup>	$U_{ij}$ / 10 <sup>-2</sup>	$D_{ij}$ / 10 <sup>-2</sup>	$U_{ij}$ / 10 <sup>-2</sup>		
BNM-INM	-0.80	0.57			-1.18	0.86	-0.89	0.71	-0.45	0.86	0.01	0.79	-2.90	0.86	-0.93	0.66	-0.65	0.73	-0.77	0.71	-0.80	1.17	-1.82	0.79	-0.43	0.83	-1.15	0.70	-0.56	1.66	-1.10	0.78	-0.64	0.79
CSIC	0.38	0.61	1.18	0.86			0.29	0.74	0.73	0.89	1.19	0.82	-1.72	0.89	0.25	0.69	0.53	0.76	0.41	0.74	0.38	1.19	-0.64	0.82	0.75	0.86	0.03	0.74	0.62	1.67	0.08	0.81	0.54	0.82
*CSIRO	0.09	0.39	0.89	0.71	-0.29	0.74			0.44	0.74	0.90	0.67	-2.01	0.74	-0.04	0.50	0.24	0.60	0.12	0.57	0.09	1.09	-0.93	0.67	0.46	0.71	-0.26	0.56	0.33	1.60	-0.21	0.65	0.25	0.67
HUT	-0.35	0.61	0.45	0.86	-0.73	0.89	-0.44	0.74			0.46	0.82	-2.45	0.89	-0.48	0.69	-0.20	0.76	-0.32	0.74	-0.35	1.19	-1.37	0.82	0.02	0.86	-0.70	0.74	-0.11	1.67	-0.65	0.81	-0.19	0.82
IRL	-0.81	0.51	-0.01	0.79	-1.19	0.82	-0.90	0.67	-0.46	0.82			-2.91	0.82	-0.94	0.61	-0.66	0.69	-0.78	0.67	-0.81	1.15	-1.83	0.75	-0.44	0.79	-1.16	0.66	-0.57	1.64	-1.11	0.74	-0.65	0.75
(KRIS)	2.10	0.64	2.90	0.86	1.72	0.89	2.01	0.74	2.45	0.89	2.91	0.82			1.97	0.69	2.25	0.76	2.13	0.74	2.10	1.19	1.08	0.82	2.47	0.86	1.75	0.74	2.34	1.67	1.80	0.81	2.26	0.82
*NIM	0.13	0.30	0.93	0.66	-0.25	0.69	0.04	0.50	0.48	0.69	0.94	0.61	-1.97	0.69			0.28	0.53	0.16	0.50	0.13	1.06	-0.89	0.61	0.50	0.66	-0.22	0.49	0.37	1.58	-0.17	0.59	0.29	0.61
NIST	-0.15	0.42	0.65	0.73	-0.53	0.76	-0.24	0.60	0.20	0.76	0.66	0.69	-2.25	0.76	-0.28	0.53			-0.12	0.60	-0.15	1.11	-1.17	0.69	0.22	0.73	-0.50	0.58	0.09	1.61	-0.45	0.67	0.01	0.69
*NPL	-0.03	0.39	0.77	0.71	-0.41	0.74	-0.12	0.57	0.32	0.74	0.78	0.67	-2.13	0.74	-0.16	0.50	0.12	0.60			-0.03	1.09	-1.05	0.67	0.34	0.71	-0.38	0.56	0.21	1.60	-0.33	0.65	0.13	0.67
NRC	0.00	1.01	0.80	1.17	-0.38	1.19	-0.09	1.09	0.35	1.19	0.81	1.15	-2.10	1.19	-0.13	1.06	0.15	1.11	0.03	1.09			-1.02	1.15	0.37	1.17	-0.35	1.09	0.24	1.85	-0.30	1.14	0.16	1.15
OFMET	1.02	0.51	1.82	0.79	0.64	0.82	0.93	0.67	1.37	0.82	1.83	0.75	-1.08	0.82	0.89	0.61	1.17	0.69	1.05	0.67	1.02	1.15			1.39	0.79	0.67	0.66	1.26	1.64	0.72	0.74	1.18	0.75
OMH	-0.37	0.57	0.43	0.83	-0.75	0.86	-0.46	0.71	-0.02	0.86	0.44	0.79	-2.47	0.86	-0.50	0.66	-0.22	0.73	-0.34	0.71	-0.37	1.17	-1.39	0.79			-0.72	0.70	-0.13	1.66	-0.67	0.78	-0.21	0.79
*PTB	0.35	0.37	1.15	0.7	-0.03	0.74	0.26	0.56	0.70	0.74	1.16	0.66	-1.75	0.74	0.22	0.49	0.50	0.58	0.38	0.56	0.35	1.09	-0.67	0.66	0.72	0.70			0.59	1.60	0.05	0.64	0.51	0.66
SMU	-0.24	1.54	0.56	1.66	-0.62	1.67	-0.33	1.60	0.11	1.67	0.57	1.64	-2.34	1.67	-0.37	1.58	-0.09	1.61	-0.21	1.60	-0.24	1.85	-1.26	1.64	0.13	1.66	-0.59	1.60			-0.54	1.63	-0.08	1.64
VNIOFI	0.30	0.49	1.10	0.78	-0.08	0.81	0.21	0.65	0.65	0.81	1.11	0.74	-1.80	0.81	0.17	0.59	0.45	0.67	0.33	0.65	0.30	1.14	-0.72	0.74	0.67	0.78	-0.05	0.64	0.54	1.63			0.46	0.74
BIPM	-0.16	0.51	0.64	0.79	-0.54	0.82	-0.25	0.67	0.19	0.82	0.65	0.75	-2.26	0.82	-0.29	0.61	-0.01	0.69	-0.13	0.67	-0.16	1.15	-1.18	0.75	0.21	0.79	-0.51	0.66	0.08	1.64	-0.46	0.74		

\* Lab the weight applied to the laboratory measurement in the calculation of the KCRV corresponds to the cutoff uncertainty  
 (Lab) the laboratory measurement is not used in the calculation of the KCRV

Table A3.3: Degrees of equivalence (blue) and bilateral degrees of equivalence (yellow)

## Appendix 4

### Formulas for Degrees of Equivalence for Weighted Mean with Cut-Off

M. Stock, BIPM

Given are the results  $x_i$  and their uncertainties  $u_i$  for  $N$  participants :  $(x_i, u_i)$  for  $i = 1, \dots, N$ .

If the key comparison reference value (KCRV) is calculated as the weighted mean, the laboratory with the smallest uncertainty will have the largest weight. If the smallest uncertainties are considered as implausible or to limit the influence of a single laboratory, the KCRV can be calculated as the weighted mean  $x_w$  with a cut-off  $u_{\min}$  (minimum) applied to the uncertainties in the calculation of the weights:

$$x_w = \sum_i w_i x_i = \frac{\sum_i \frac{x_i}{v_i^2}}{\sum_j \frac{1}{v_j^2}} \quad \text{with } w_i \equiv \frac{\frac{1}{v_i^2}}{\sum_j \frac{1}{v_j^2}} \quad \text{and}$$

$$v_i = \max(u_i, u_{\min}).$$

In the case that the uncertainty  $u_i$  is smaller than the cut-off, it is replaced by the cut-off  $u_{\min}$  in the calculation of the weight of laboratory  $i$ . The adjusted uncertainties  $v_i$  are, however, not considered as the real uncertainty of laboratory  $i$  in the following uncertainty calculation. The uncertainty  $u(x_i)$  is still  $u_i$ . This in some sense is a choice, but it is compatible with the decision of the CCPR to apply the cut-off only in the calculation of the reference value for CCPR-K3.b but to use the  $u_i$ 's in the tables and graphs showing the final results.

Under the assumption that all  $x_i$  are uncorrelated, the uncertainty of the weighted mean is given by

$$u^2(x_w) = \frac{\sum_i \frac{u_i^2}{v_i^4}}{\left( \sum_j \frac{1}{v_j^2} \right)^2} ,$$

If all  $v_i = u_i$ , that is if no cut-off is applied, the result becomes identical to the well known result

$$u^2(x_w) = \frac{1}{\sum_j \frac{1}{u_j^2}} .$$

The deviation  $d_i$  from the reference value is defined by

$$d_i = x_i - x_w = x_i - \frac{\sum_k \frac{x_k}{v_k^2}}{\sum_j \frac{1}{v_j^2}} = \left( 1 - \frac{\frac{1}{v_i^2}}{\sum_j \frac{1}{v_j^2}} \right) x_i - \frac{\sum_{k \neq i} \frac{x_k}{v_k^2}}{\sum_j \frac{1}{v_j^2}}$$

Again, if all the  $x_i$  are uncorrelated, the uncertainty of  $d_i$  is

$$\begin{aligned} u^2(d_i) &= \left( 1 - \frac{\frac{1}{v_i^2}}{\sum_j \frac{1}{v_j^2}} \right)^2 u_i^2 + \frac{\sum_{k \neq i} \frac{u_k^2}{v_k^4}}{\left( \sum_j \frac{1}{v_j^2} \right)^2} = u_i^2 + u^2(x_w) - \frac{\frac{2}{v_i^2}}{\sum_j \frac{1}{v_j^2}} u_i^2 \\ &= u_i^2 - u^2(x_w) + 2 u^2(x_w) \left[ 1 - \frac{u_i^2}{v_i^2} \frac{\sum_j \frac{1}{v_j^2}}{\sum_k \frac{u_k^2}{v_k^4}} \right] \end{aligned}$$

The last term describes the effect of the adjusted uncertainties. If no cut-off is applied (all  $v_i = u_i$ ), the term in the square brackets becomes zero and we obtain

$$u^2(d_i) = u_i^2 - u^2(x_w)$$

which is the formula already known for the weighted mean without cut-off.

**Summary:**

Uncertainty of reference value: 
$$u^2(x_w) = \frac{\sum_i \frac{u_i^2}{v_i^4}}{\left( \sum_j \frac{1}{v_j^2} \right)^2}$$

Uncertainty of deviation: 
$$u^2(d_i) = u_i^2 + u^2(x_w) - \frac{\frac{2}{v_i^2}}{\sum_j \frac{1}{v_j^2}} u_i^2$$