

**Final Report on**  
**Supplementary Comparison APMP.M.P-S2**  
**(Bilateral comparison)**  
**at a nominal pressure of 0.05 Pa**

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**Abstract.** A supplementary bilateral comparison was carried out to compare two different pressure standards at a nominal pressure of 0.05 Pa. The two standards used different principles for generating the nominal pressure; one based on the static expansion method (NPL, India), and the other based on the orifice flow or dynamic expansion method (NIST, USA). The transfer standard consisted of a pair of spinning rotor gauges whose accommodation coefficients were measured at a nominal pressure of 0.05 Pa at the two laboratories. The nominal pressure value for the comparison was chosen because (i) it can be easily generated by the respective standards with high accuracy and (ii) it had not been covered in the earlier two key comparisons, namely, CCM.P-K3 and CCM.P-K4, which the two NMIs had participated in. NPLI served as the pilot lab, and provided the stainless steel rotors; each lab used its own thimbles and electronics. The rotors were hand-carried from NPLI to NIST at atmospheric pressure in small, insulation-packed glass vials to prevent the rotors from moving during transit; otherwise no special transit precautions were taken.

The pressure value of 0.05 Pa generated by the NPLI and NIST showed good agreement within their combined uncertainties proving thereby that the two standards were equivalent to each other for the pressure of 0.05 Pa.

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## 1. Introduction

The National Physical Laboratory of India (NPLI) maintains three primary vacuum standards to generate absolute pressures in the range from 130 kPa to  $10^{-6}$  Pa. First is the primary standard based on the principle of ultrasonic interferometer manometer which is used to generate pressures in the range 130 kPa to about 1 Pa. Pressures in the range 1000 Pa to  $10^{-4}$  Pa are generated by the NPLI by means of their Static Expansion System (SES) while the Orifice flow primary high vacuum standard is used to produce vacua in the range  $10^{-2}$  Pa to  $10^{-6}$  Pa. NPLI has in the past participated in a number of comparisons, both key comparisons under the aegis of CCM [1-4], and bilateral comparisons [5], from time to time, in order to establish the international compatibility of the three standards. NPLI has also conducted in-house comparisons to establish the mutual compatibility of their three primary vacuum standards in the overlapping pressure region [6-8].

Similarly, the National Institute of Standards and Technology, (NIST), USA also maintains a number of primary standards to cover the pressure scale from 130 kPa to  $10^{-7}$  Pa, namely, (i) ultrasonic interferometer manometer and (ii) orifice flow technique. NIST has been the pilot laboratory for two key comparisons CCM.P-K3 and CCM.P-K4 [2]. In addition, NIST has participated in the CCM.P-K9 [3] and CCM.P-K10 [1] key comparisons to establish the degree of equivalence of their standards.

Nominal pressures of 0.05 Pa are generated (i) at the NPLI by their SES using the method of single stage expansion and (ii) at the NIST by their mid-range orifice flow standard. NPLI has recently made a fresh determination of the volume ratio of their SES using two resonance silicon gauges of different full-scale ranges to measure the initial and the final pressures. NPLI has also recently undergone a peer review of their CMCs. NIST has, in the meantime, moved into their newly built Advanced Metrology Laboratory building which was designed to give high temperature stability and protection from noise and vibration. Following these developments, it was decided by the two laboratories to inter-compare their respective standards at a nominal pressure of 0.05 Pa using a pair of spinning rotor gauges as transfer standards, to establish their degree of equivalence. This value of the nominal pressure was chosen since (i) it can be easily generated by their respective standards with high accuracy and (ii) it had never been covered in the earlier two comparisons, namely CCM.P-K3 and CCM.P-K4. This comparison is listed as a supplementary comparison APMP.M.P-S2 in the BIPM database with NPLI as the pilot laboratory.

## 2. Participating laboratories and their standards

Table 1 lists the two participating laboratories and characteristics of their standards. The NPL pressure standard (SES) is based on the static expansion method and has been well described in literature [3, 5-8]. It is a two-stage expansion system capable of generating pressures in the range 1000 Pa down to  $10^{-4}$  Pa. It consists of five known volumes, three small volumes of nominally 0.384 L, 0.025 L and 0.025 L respectively and two expansion chambers, 72 L and 24 L nominally. In this primary standard, gas at a known relatively high pressure is enclosed in one of the small volumes and then allowed to expand into a previously evacuated, larger volume (under isothermal conditions), resulting in a pressure drop. Using the initial gas pressure, the ratio of the two volumes involved, and the ratio of the beginning and ending temperatures, the final pressure after expansion is calculated from the equation of state for an ideal gas (with appropriate virial corrections)

Table 1. List of participants and the standards used for the calibration of the transfer standards.

Laboratory	Standard	Type	Traceable to	CMC listed
National Physical Laboratory, India	Static expansion system	Primary	Independent	Yes
National Institute of Standards and Technology, USA	Orifice flow system	Primary	Independent	Yes

The primary standard of the NIST is an Orifice Flow system called Mid-range transition flow standard [9] in which pressures are generated by making use of the well-established orifice flow technique. This standard covers the range from 30 pascals to  $10^{-4}$  Pa and is routinely used for calibration of spinning rotor gauges.

### 3. Transfer standards

Two spinning rotor gauges (SRG) were chosen as transfer standard for the comparison. The SRG [10] has been widely employed as a transfer standard due to its measurement accuracy and long-term and transport stability [3-5, 7, 11-12]. Two devices were used in order to further reduce the influence of transport-instabilities, to produce redundancy, and to increase the accuracy of the comparison.

Only the rotor balls were provided as the transfer standard item. The thimbles with connecting flange for housing the rotors, and the electronic controllers were provided by the respective participants, as it is believed that the controllers have no effect on the results of the calibration. The thimbles used at the two laboratories had the same nominal OD of 8 mm, and were considered to be identical.

The rotors needed for the comparison were provided by NPLI. Rotor 1 was a smooth stainless steel ball; with a nominal diameter of 4.5 mm. Rotor 2 was a SS rotor with an etched surface with a nominal diameter of 4.489 mm.

Table 2 Transfer standards used for the comparison.

Transfer standard	Rotor 1	Rotor 2
Designation	NPL-0	NPL-1
Material	SS, smooth	SS etched
Nominal diameter	4.500 mm	4.489 mm
Density	7700 kg/m <sup>3</sup>	7743 kg/m <sup>3</sup>

Both these rotors have been used at the NPLI since 1987 and the trend for their accommodation coefficients are well known. One of the rotors, NPL-1, has additionally been used on two occasions to participate in comparisons with IMGIC, Italy.

#### 4. Calibration constant

The value to be calibrated by each laboratory  $j$  for the specified pressure (0.05 Pa) for each rotor  $i$  was the effective accommodation coefficient  $\sigma_{ij}$ , which is mainly determined by the tangential momentum transfer coefficient of the gas molecules to the rotor, and partly by the energy accommodation factor [10]

$\sigma_{ij}$  was determined by the following equation:

$$\sigma_{ij} = \frac{d_i \rho_i}{20 p_{rj}} \sqrt{\frac{8\pi RT_j}{M}} \left( \left( \frac{\dot{\omega}}{\omega} \right)_{ij} - RD_{ij}(\omega) \right) \quad (1)$$

$$\sigma_{ij} = \frac{d_i \rho_i}{20 p_{rj}} \sqrt{\frac{8\pi RT_j}{M}} (DCR_{ij} - RD_{ij}(\omega)) \quad (2)$$

where  $p_{rj}$  is the generated pressure in the standard,  $T_j$  the temperature of the gas in the calibration chamber,  $d_i$  and  $\rho_i$  are the diameter and density of the rotor  $i$ ,  $M$  is the molecular mass of nitrogen, and the relative deceleration rate of the rotor frequency, DCR, is defined as

$$DCR = \dot{\omega} / \omega$$

The quantity  $RD_{ij}$  is a pressure independent residual drag, caused by eddy current losses in the surrounding metal structures and the rotor itself.

The residual drag is generally a function of  $\omega$ ,  $RD_{ij}(\omega)$ , so that it was required that, whenever  $\sigma_{ij}$  was determined, the value of  $\omega$  also had to be measured in order to subtract the correct value for  $RD_{ij}(\omega)$  in Eq. (1). For measuring  $RD_{ij}(\omega)$  the following procedures were adopted:

**At NPL:**  $RD_{ij}(\omega)$  was determined during the course of the calibrations by pumping down the vacuum system to residual pressure conditions both before and after each target pressure point measurement. The mean of the two RD values was subtracted from the DCR value between these  $RD(\omega)$  values, i.e.,

$$\sigma_{iNPL} = \frac{d_i \rho_i}{20 p_{rNPL}} \sqrt{\frac{8\pi RT_{NPL}}{M}} \left( DCR_{iNPL} - \left( \frac{RD_{iNPLb}(\omega) + RD_{iNPLa}(\omega)}{2} \right) \right) \quad (3)$$

where the subscripts  $b$  and  $a$  refer to “before” and “after” each target pressure point measured. This procedure was applicable as long as the frequency showed a one-way trend, i.e., either a rise or a fall and also the RD did not show a pronounced linear dependence on the frequency.

**AT NIST:** The RD was plotted as a function of the rotor frequency over long periods of time, to cover the full frequency range, before starting the calibrations at the ultimate pressure of the mid-range standard, typically in the range of  $10^{-6}$  Pa. A linear least squares fit was applied to obtain the

function  $RD_{iNIST}(\omega)$  and the standard deviation of the fit was used to calculate the standard uncertainty in the residual drag.

## 5. Organization of the comparison and chronology

Measurements were first performed at NPLI, the pilot laboratory. The two rotors were then hand-carried to NIST for calibration measurements to be performed. They were then brought back to NPLI and measurements were repeated. The act of hand-carrying the rotors from and to the pilot laboratory was adopted in order to reduce the effects of transport related instabilities. Each of the rotors was wrapped in aluminum foil and kept in a plastic box lined with soft sponge during storage and transit.

The sequence of measurements at the two laboratories is shown in Table 3. The suffix “b” refers to measurements made at NPLI *before* hand carrying the rotors to NIST while the suffix “a” refers to those made *after* their return from NIST.

Table 3 Chronology of measurements

Calibrating Laboratory and the cycle number	Date	Remarks
NPL-b(1) & NPL-b(2)	Sep. 30 and Oct. 3, 2005	Hand carried to NIST afterwards
NIST-1 and NIST-2	Nov. 4 and 7, 2005	Hand carried to NPLI afterwards
NPL-a(1) & NPL-a(2)	Dec. 16 and 23, 2005	Comparison concluded

## 6. Calibration procedure and results to be reported

Each laboratory calibrated the two rotors ten times at a nominal pressure of 0.05 Pa on two different days. Each set of 10 readings made on a day was called one measurement cycle. Each reading consisted of 10 printouts of the SRG indications at suitable measuring times. In addition, the pilot Laboratory NPLI performed two more cycles of calibration after measurements were made at NIST. In this way, NPLI made four cycles in all while NIST made two cycles. As stated earlier, the rotors were carried to NIST without their thimbles and were loaded into thimbles provided by NIST and mounted on the NIST mid-range orifice flow standard. Neither the NPLI standard nor the NIST standard was baked.

If the target pressure was not reached accurately, then  $\sigma_{ij}$  could be calculated from the following expression:

$$\sigma_{ij} = (\sigma_{ij})_{\text{det}} + (p_t - p_{rj}) \cdot m_{iNPL} \quad (4)$$

Here,  $p_t$  is the target pressure of 0.05 Pa,  $\sigma_{ij}$  is the value determined at  $p_{rj}$  and  $m_{iNPL}$  is the slope of the fit of  $\sigma_i$  over a pressure range of 0.1 to 1.0 Pa determined in a supplementary experiment at the pilot Laboratory. At the end of the comparison, the values of  $\sigma_{ij}$  for the individual measurements at the nominal pressure and the standard uncertainty  $u(p_{rj})$  of  $p_{rj}$  were provided by each laboratory.

## 7. Uncertainties of primary standards

The relative standard uncertainty  $\frac{u(p_{rj})}{p_{rj}}$  (Type B) evaluated at a coverage factor of 1 ( $k=1$ ) for NPLI was 0.002 while that for NIST was 0.0015.

## 8. Uncertainty of reported measured values

Eqn. (2) can be used to write an expression for the relative standard uncertainty in the value of  $\sigma_{ij}$  as follows [11, 12]: (Reference Jousten, et. al Metrologia 2004 Euromet comparison)

$$\frac{u(\sigma_{ij})}{\sigma_{ij}} = \sqrt{\left(c_1 \frac{u(T_{ij})}{T_{ij}}\right)^2 + \left(c_2 \frac{u(p_{rj})}{p_{rj}}\right)^2 + \left(c_3 \frac{u(DCR_{ij})}{DCR_{ij}}\right)^2 + \left(c_4 \frac{u(RD_{ij})}{RD_{ij}}\right)^2 + \left(\frac{s.d.(\sigma_{ij})}{\sqrt{n}} \frac{1}{\sigma_{ij}}\right)^2} \quad (5)$$

where  $c_i$  are the sensitivity coefficients defined as

$$c_1 = 1/2 \quad (6)$$

$$c_2 = 1 \quad (7)$$

$$c_3 = \frac{DCR_{ij}}{DCR_{ij} - RD_{ij}} \quad (8)$$

$$c_4 = \frac{RD_{ij}}{DCR_{ij} - RD_{ij}} \quad (9)$$

When  $RD_{ij}$  is written as the mean of the two  $RD_{ij}$  terms, one before and the other after the  $DCR_{ij}$  measurement, Eqn. (5) takes the form

$$\frac{u(\sigma_{ij})}{\sigma_{ij}} = \sqrt{\left(c_1 \frac{u(T_{ij})}{T_{ij}}\right)^2 + \left(c_2 \frac{u(p_{rj})}{p_{rj}}\right)^2 + \left(c_3 \frac{u(DCR_{ij})}{DCR_{ij}}\right)^2 + \left(c_4 \frac{u(RD_{ijb})}{RD_{ijb}}\right)^2 + \left(c_5 \frac{u(RD_{ija})}{RD_{ija}}\right)^2 + \left(\frac{s.d.(\sigma_{ij})}{\sqrt{n}} \frac{1}{\sigma_{ij}}\right)^2} \quad (10)$$

Here  $c_3$  is modified to

$$c_3 = \frac{DCR_{ij}}{(DCR_{ij} - (RD_{ijb} + RD_{ija})/2)} \quad (11)$$

with

$$c_4 = \frac{RD_{ijb}}{2(DCR_{ij} - (RD_{ijb} + RD_{ija})/2)} \quad (12)$$

and

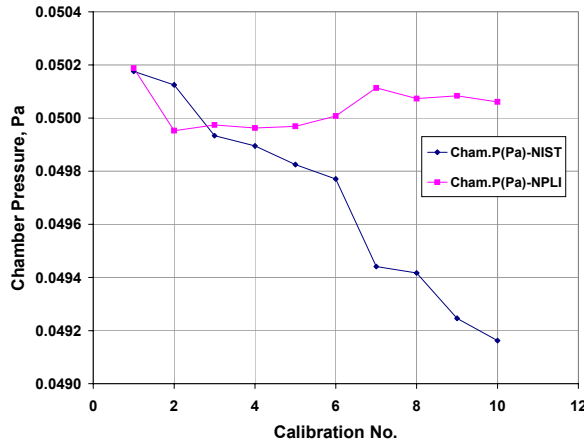
$$c_5 = \frac{RD_{ija}}{2(DCR_{ij} - (RD_{ijb} + RD_{ija})/2)} \quad (13)$$

The last term under the radical sign in Eqn. (5 and 10) accounts for the standard deviation in the 10 readings of  $\sigma$  in a particular calibration cycle.

## 9. Results

Figure 1 shows a typical pattern of pressures generated on the two standards. One can notice that while in the case of NPLI the pressures generated for the 10 readings show a random spread, those for NIST show a linear dependence. This is easy to understand since in the case of NPLI the standard consisted of the Static expansion system and one could always expect to obtain the same final pressure in the calibration chamber if one started with the same initial pressure. In the case of NIST, the procedure is a little different. The constant pressure flowmeter of the NIST standard is initially filled with gas at a predetermined nominal pressure which decreases with successive calibrations. Nevertheless, the fall in pressure is so small that in the present case it is estimated, that the difference in the values of  $\sigma_{\text{NIST}}$  obtained from the actual values of pressures differs by not

**Fig. 1. Typical pattern of Chamber pressure plotted against the measurement number at the two NMIs.**



more than a few ppm from those calculated using Eqn. (4). From the two measurements of the generated pressure and the indicated pressure, the corresponding sigma values for the two rotors were calculated using Eqn. (2 or 3). These results are listed in Tables 4 through 9. Also listed in the Tables are the corresponding standard uncertainties  $u(\sigma_{ij})$  calculated from Eqn. (5 or 10). The uncertainties  $u'(\sigma_{ij})$  listed in these tables are those calculated by omitting the standard uncertainty in  $p_r$ . This will be discussed in a latter section. Measurements of  $\sigma_i$  were made at 296 K at NPLI while the laboratory temperature at NIST was 294.8 K.

Table 10 was constructed using the Tables 4 through 9. The mean value of the gauge coefficient of a rotor for a particular cycle was obtained from the mean of the ten readings of that particular day. The highest standard uncertainty associated with any of these ten readings was assigned as the standard uncertainty of the mean of that particular cycle. The mean value of the pilot lab calibrations, before and after measurements at NIST, was taken as the reference value of the effective accommodation coefficient for comparing the standards of the two labs. Figs. 1 and 2 show plots of the six calibration cycles made at both the laboratories along with their standard uncertainties ( $k = 1$ ). Also plotted in these figures is the reference mean obtained from the four calibrations performed at the pilot laboratory, NPLI. It can be seen from Table 10 and figures 2 and 3 that the values of sigma

obtained for the two rotors at the two laboratories are in good agreement. The components of the relative standard uncertainty in sigma for the rotor NPL-0 corresponding to the two participating laboratories are shown in Figs. 4 and 5.

Table 4. The values of  $\sigma_1$  of Rotor 1 (NPL-0) and the associated uncertainties for two calibration cycles performed at NPLI before NIST calibrations. The suffixes b1 and b2 refer to the two calibration cycles “before” measurements at NIST.

$P_t$ $P_a$	NPL-0(b1)			NPL-0(b2)		
	$\sigma_1$	$u(\sigma_1)$	$u'(\sigma_1)$ omitting $u(p_r)$	$\sigma_1$	$u(\sigma_1)$	$u'(\sigma_1)$ omitting $u(p_r)$
	Sep. 30, 2005			Oct. 3, 2005		
0.05	1.0348	0.00209	0.00028	1.0377	0.00211	0.00036
0.05	1.0351	0.00208	0.00023	1.0360	0.00210	0.00034
0.05	1.0343	0.00208	0.00022	1.0364	0.00210	0.00033
0.05	1.0345	0.00211	0.00040	1.0355	0.00212	0.00046
0.05	1.0350	0.00208	0.00019	1.0364	0.00209	0.00028
0.05	1.0338	0.00210	0.00035	1.0354	0.00210	0.00033
0.05	1.0349	0.00208	0.00020	1.0361	0.00210	0.00033
0.05	1.0356	0.00216	0.00060	1.0357	0.00211	0.00038
0.05	1.0355	0.00209	0.00026	1.0355	0.00210	0.00035
0.05	1.0350	0.00209	0.00029	1.0344	0.00213	0.00049

Table 5. The values of  $\sigma_1$  of Rotor 1 (NPL-0) and the associated uncertainties for two calibration cycles performed at NPLI after NIST calibrations. The suffixes a1 and a2 refer to the two calibration cycles “after” measurements at NIST.

$P_t$ $P_a$	NPL-0(a1)			NPL-0(a2)		
	$\sigma_1$	$u(\sigma_1)$	$u'(\sigma_1)$ omitting $u(p_r)$	$\sigma_1$	$u(\sigma_1)$	$u'(\sigma_1)$ omitting $u(p_r)$
	Dec. 16, 2005			Dec. 23, 2005		
0.05	1.0348	0.00229	0.00099	1.0336	0.00209	0.00030
0.05	1.0328	0.00237	0.00117	1.0337	0.00209	0.00030
0.05	1.0339	0.00220	0.00075	1.0335	0.00210	0.00039
0.05	1.0329	0.00215	0.00061	1.0332	0.00215	0.00058
0.05	1.0331	0.00238	0.00118	1.0330	0.00211	0.00044
0.05	1.0341	0.00221	0.00077	1.0331	0.00212	0.00045
0.05	1.0329	0.00218	0.00069	1.0325	0.00211	0.00044
0.05	1.0337	0.00217	0.00066	1.0324	0.00209	0.00033
0.05	1.0340	0.00214	0.00053	1.0323	0.00209	0.00033
0.05	1.0335	0.00210	0.00037	1.0319	0.00213	0.00053

Table 6. The NIST values of  $\sigma_1$  of Rotor 1 (NPL-0) and the associated uncertainties for two calibration cycles performed between calibrations at NPLI. The suffixes 1 and 2 refer to the two calibration cycles measurements at NIST.

<b>P<sub>t</sub></b>	<b>NPL-0(1)</b>			<b>NPL-0(2)</b>		
<b>Pa</b>	$\sigma_1$	<b>u(<math>\sigma_1</math>)</b>	<b>u'(<math>\sigma_1</math>) omitting u(p<sub>r</sub>)</b>	$\sigma_1$	<b>u(<math>\sigma_1</math>)</b>	<b>u'(<math>\sigma_1</math>) omitting u(p<sub>r</sub>)</b>
	Nov. 4, 2005			Nov. 7, 2005		
0.05	1.0340	0.001742	0.00079	1.0339	0.001738	0.00079
0.05	1.0336	0.001741	0.00079	1.0346	0.001740	0.00079
0.05	1.0347	0.001745	0.00080	1.0349	0.001740	0.00079
0.05	1.0345	0.001743	0.00079	1.0344	0.001740	0.00079
0.05	1.0345	0.001743	0.00079	1.0349	0.001743	0.00079
0.05	1.0338	0.001742	0.00079	1.0346	0.001740	0.00079
0.05	1.0346	0.001744	0.00080	1.0337	0.001738	0.00079
0.05	1.0331	0.001740	0.00079	1.0345	0.001739	0.00079
0.05	1.0342	0.001742	0.00079	1.0345	0.001740	0.00079
0.05	1.0340	0.001743	0.00080	1.0344	0.001740	0.00079

Table 7. The values of  $\sigma_2$  of Rotor 2 (NPL-1) and the associated uncertainties for two calibration cycles performed at NPLI before NIST calibrations. The suffixes b1 and b2 refer to the two calibration cycles “before” measurements at NIST.

<b>P<sub>t</sub></b>	<b>NPL-1(b1)</b>			<b>NPL-1(b2)</b>		
<b>Pa</b>	$\sigma_2$	<b>u(<math>\sigma_2</math>)</b>	<b>u'(<math>\sigma_2</math>) omitting u(p<sub>r</sub>)</b>	$\sigma_2$	<b>u(<math>\sigma_2</math>)</b>	<b>u'(<math>\sigma_2</math>) omitting u(p<sub>r</sub>)</b>
	Sep. 30, 2005			Oct. 3, 2005		
0.05	1.1136	0.00225	0.00032	1.1163	0.00227	0.00036
0.05	1.1137	0.00224	0.00028	1.1159	0.00226	0.00030
0.05	1.1133	0.00223	0.00013	1.1153	0.00226	0.00032
0.05	1.1137	0.00226	0.00035	1.1151	0.00228	0.00041
0.05	1.1135	0.00223	0.00010	1.1151	0.00225	0.00029
0.05	1.1137	0.00225	0.00033	1.1144	0.00226	0.00034
0.05	1.1138	0.00223	0.00007	1.1141	0.00226	0.00032
0.05	1.1137	0.00225	0.00032	1.1142	0.00227	0.00038
0.05	1.1136	0.00224	0.00019	1.1138	0.00226	0.00033
0.05	1.1135	0.00224	0.00025	1.1129	0.00229	0.00047

Table 8. The values of  $\sigma_2$  of Rotor 2 (NPL-1) and the associated uncertainties for two calibration cycles performed at NPLI after NIST calibrations. The suffixes a1 and a2 refer to the two calibration cycles “after” measurements at NIST.

$P_t$	NPL-1(a1)			NPL-1(a2)		
Pa	$\sigma_2$	$u(\sigma_2)$	$u'(\sigma_2)$ omitting $u(p_r)$	$\sigma_2$	$u(\sigma_2)$	$u'(\sigma_2)$ omitting $u(p_r)$
	Dec. 16, 2005			Dec. 23, 2005		
0.05	1.1148	0.00224	0.00022	1.11389	0.00225	0.00035
0.05	1.1142	0.00231	0.00062	1.11390	0.00226	0.00035
0.05	1.1148	0.00225	0.00030	1.11379	0.00227	0.00045
0.05	1.1149	0.00229	0.00051	1.11360	0.00232	0.00064
0.05	1.1153	0.00225	0.00033	1.11332	0.00228	0.00049
0.05	1.1149	0.00225	0.00033	1.11322	0.00228	0.00050
0.05	1.1149	0.00230	0.00057	1.11269	0.00228	0.00049
0.05	1.1143	0.00228	0.00050	1.11262	0.00226	0.00038
0.05	1.1146	0.00227	0.00041	1.11247	0.00226	0.00038
0.05	1.1147	0.00224	0.00026	1.11150	0.00230	0.00059

Table 9. The NIST values of  $\sigma_2$  of Rotor 2 (NPL-1) and the associated uncertainties for two calibration cycles performed between calibrations at NPLI. The suffixes 1 and 2 refer to the two calibration cycle measurements at NIST.

$P_t$	NPL-1(1)			NPL-1(2)		
Pa	$\sigma_2$	$u(\sigma_2)$	$u'(\sigma_2)$ omitting $u(p_r)$	$\sigma_2$	$u(\sigma_2)$	$u'(\sigma_2)$ omitting $u(p_r)$
	Nov. 4, 2005			Nov. 7, 2005		
0.05	1.1162	0.00188	0.00086	1.1153	0.00188	0.00086
0.05	1.1155	0.00188	0.00086	1.1168	0.00188	0.00086
0.05	1.1164	0.00188	0.00086	1.1172	0.00188	0.00086
0.05	1.1165	0.00188	0.00086	1.1168	0.00189	0.00086
0.05	1.1154	0.00188	0.00086	1.1172	0.00189	0.00087
0.05	1.1158	0.00188	0.00086	1.1163	0.00189	0.00087
0.05	1.1168	0.00188	0.00086	1.1159	0.00188	0.00086
0.05	1.1151	0.00188	0.00085	1.1169	0.00188	0.00086
0.05	1.1162	0.00188	0.00086	1.1169	0.00189	0.00086
0.05	1.1162	0.00188	0.00086	1.1158	0.00188	0.00086

Table 10. Condensed calibration results obtained by using Tables 4 through 9.

		Mean	Reference		$u'(\sigma_{NPL-0})$	Mean	Reference		$u'(\sigma_{NPL-1})$
	Cycle	$\sigma_{NPL-0}$	Mean $\sigma_{NPL-0}$	$u(\sigma_{NPL-0})$	omitting $u(p_r)$	$\sigma_{NPL-1}$	Mean $\sigma_{NPL-1}$	$u(\sigma_{NPL-1})$	omitting $u(p_r)$
09/30/05	NPL-b1	1.03486	1.03431	0.00216	0.00060	1.11362	1.11405	0.00226	0.00035
10/03/05	NPL-b2	1.03589	1.03431	0.00213	0.00049	1.11472	1.11405	0.00229	0.00047
11/04/05	NIST-1	1.03408	1.03431	0.00174	0.00080	1.11601	1.11405	0.00188	0.00086
11/07/05	NIST-2	1.03444	1.03431	0.00174	0.00079	1.11651	1.11405	0.00189	0.00087
12/16/06	NPL-a1	1.03356	1.03431	0.00238	0.00118	1.11475	1.11405	0.00231	0.00062
12/23/06	NPL-a2	1.03293	1.03431	0.00215	0.00058	1.11310	1.11405	0.00232	0.00064

Fig. 2. Plot of calibration results for rotor NPL-0 (the error bars denote standard uncertainties (k=1))

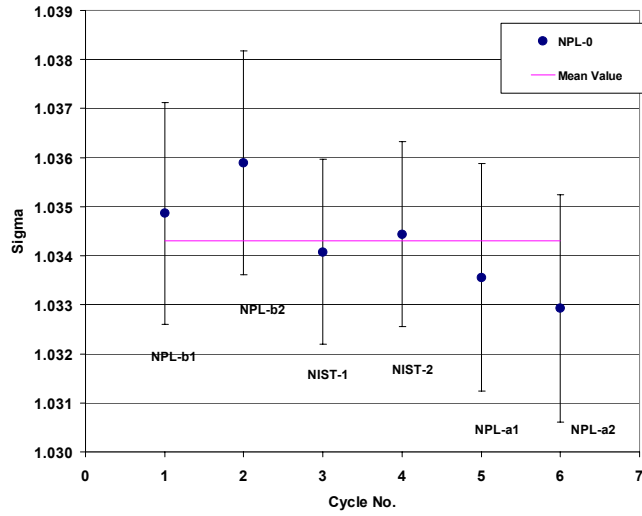


Fig. 3. Plot of calibrations results for rotor NPL-1

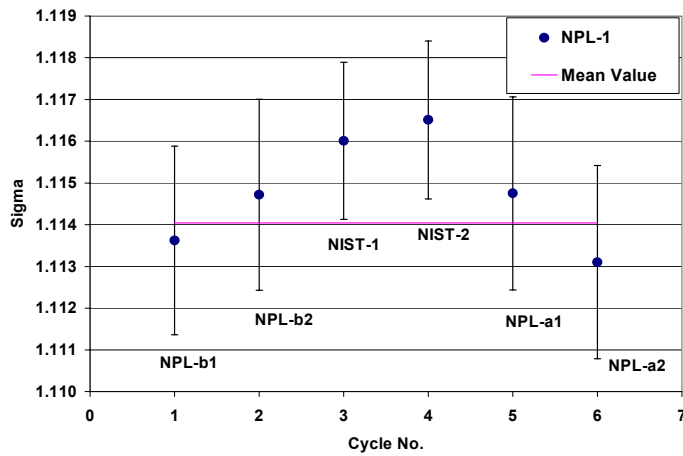


Fig. 4. Component rel. standard uncertainties in sigma for NPL-0 measured at NPLI

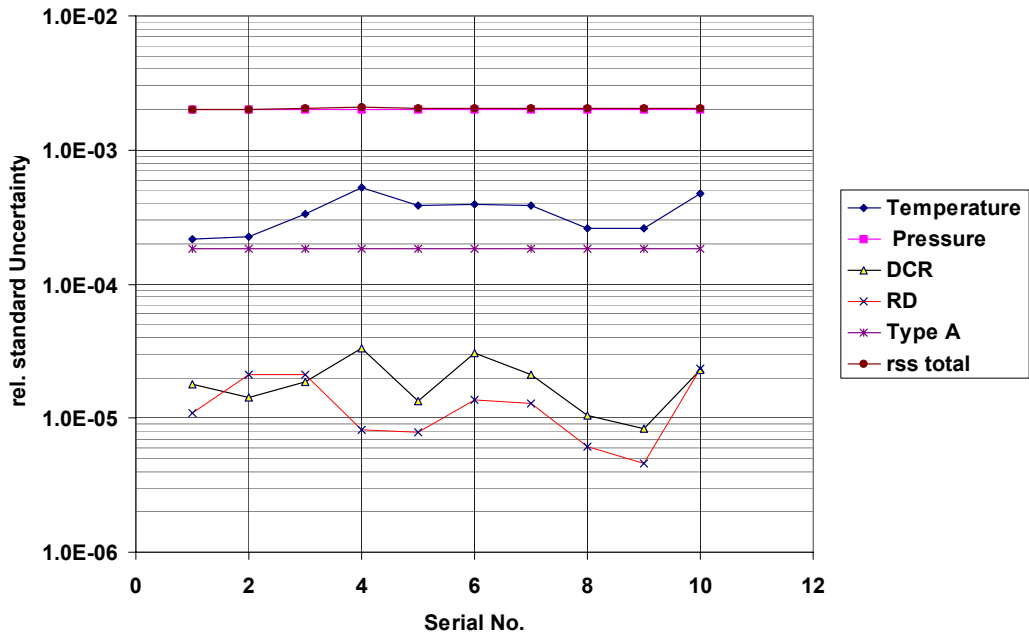
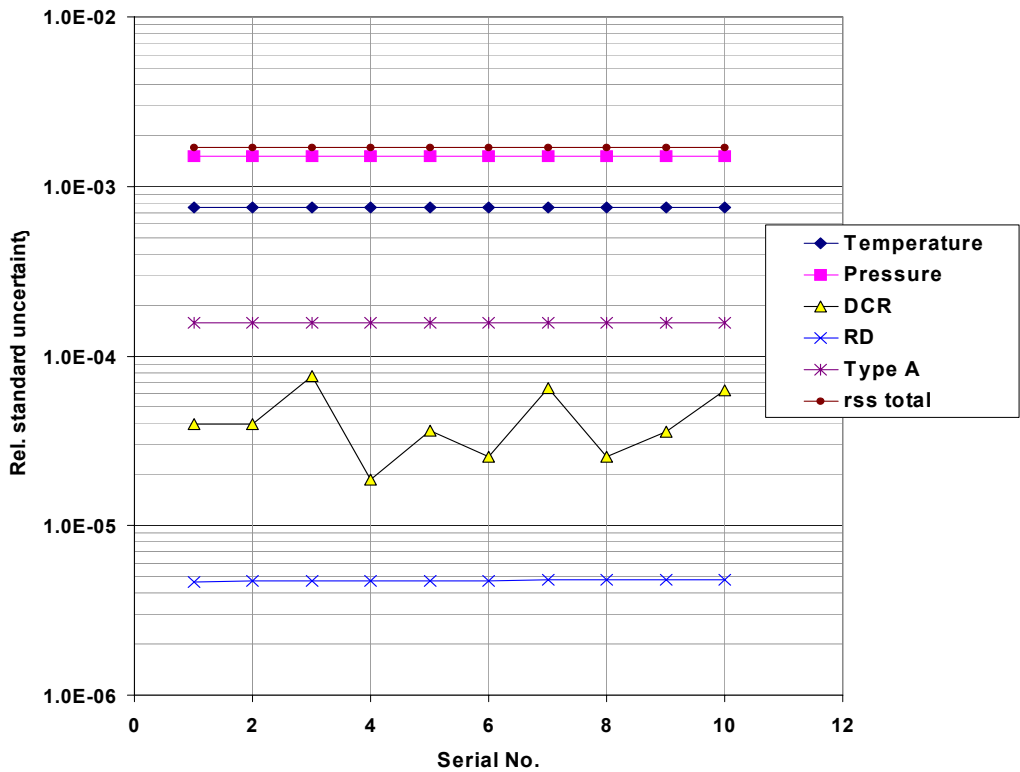


Fig. 5. Component relative standard uncertainties in  $\sigma_0$  for NPL-0 measured at NIST



## 10. Data reduction and evaluation of degree of equivalence

Using the above results we can now compare the generated pressures of the two standards following Jousten et. al. [12]. For each laboratory, and for each SRG  $i$  a value of indicated pressure  $p_{ij}$  for a common nominal target pressure  $p_t$  can be calculated using the equation:

$$p_{ij} = p_t \cdot \sigma_{ij} \quad i = 1, 2 \quad j = 1, 2 \quad (14)$$

$\sigma_{ij}$  denotes the mean accommodation coefficient of SRG  $i$  as determined by laboratory  $j$  ( $j = 1$  for NPLI and 2 for NIST).

Here the values of  $\sigma_{ij}$  are used to predict gauge readings that would be observed when the two standards are set to generate pressures of the same value at the same time. The difference in the gauge readings is an indicator of the difference between the true pressures realized by the different standards for the common nominal calculated pressure. This latter difference between calculated pressures, when the standards are set to produce exactly the same transfer gauge reading near the target pressure, is to a very good approximation equal to the difference in the predicted gauge readings but of opposite sign.

Since  $p_t$  is simply a numerical value without uncertainty, the uncertainty in  $p_{ij}$  is calculated from the following equation:

$$u(p_{ij}) = p_{ij} \frac{u(\sigma_{ij})}{\sigma_{ij}} \quad (15)$$

Table 11 lists the values of  $p_{ij}$  and  $u(p_{ij})$  obtained from Eqns. 14 and 15.

Table 11. The predicted gauge readings and their uncertainties for rotor 1 (NPL-0) and rotor 2 (NPL-1).

	$P_t, \text{ Pa}$	$p_{\text{NPL}}$	$u(p_{\text{NPL}})$	$p_{\text{NIST}}$	$u(p_{\text{NIST}})$
Rotor 1	0.05	5.1716E-02	1.1898E-04	5.1713E-02	8.7000E-05
Rotor 2	0.05	5.5702E-02	1.1585E-04	5.5813E-02	9.4444E-05

The degree of equivalence between the two institutes can be tested using

$$d_i = p_{\text{NIST}} - p_{\text{NPL}} \quad (16)$$

for the target pressure and each transfer standard  $i$ . The uncertainty  $u(d_i)$  is given by

$$u(d_i) = \sqrt{u^2(p_{\text{NIST}}) + u^2(p_{\text{NPL}})}. \quad (17)$$

The quantity  $E_n$  is calculated as

$$E_{ni} = \frac{d_i}{2u(d_i)} \quad (18)$$

Equivalence is assumed with the reference value if  $|E_{ni}| \leq 1$ . The values of  $d_i$ ,  $u(d_i)$  and  $E_n$  calculated for the two rotors for the target pressure of 0.05 Pa have been listed in Table 12. Since  $E_n < 1$  for both of the rotors, equivalence is implied in the standards maintained by the two participating laboratories.

Table 12.  $E_n$  values calculated for the two rotors.

$P_t$	Rotor	$d_i$	$u(d_i)$	$E_n$
0.05 Pa	NPL-0	-2.5913E-06	1.4740E-04	-0.0088
0.05 Pa	NPL-1	1.1068E-04	1.4947E-04	0.3702

One can also combine the two results and get a single value of  $E_n$  as follows:  
We define

$$r_i = \frac{p_{iNIST}}{p_{iNPL}} \quad (19)$$

with

$$u(r_i) \approx \sqrt{\left(\frac{u(p_{iNIST})}{p_{iNIST}}\right)^2 + \left(\frac{u(p_{iNPL})}{p_{iNPL}}\right)^2} \quad (20)$$

Table 13 shows the values of  $r_i$  and  $u(r_i)$  for the two rotors.

Table 13. The ratios  $r_i = p_{iNIST}/p_{iNPL}$  and their uncertainties determined by Eq. (19) and (20).

$P_t$ , Pa	$r_1$	$r_2$	$u(r_1)$	$u(r_2)$	$u'(r_1)$	$u'(r_2)$
0.05	0.9999	1.0020	0.0029	0.0027	0.0014	0.0010

Since the same standards  $j$  were used to determine  $\sigma_{1j}$  and  $\sigma_{2j}$ ,  $r_1$  and  $r_2$  are correlated. According to Jousten et. al. [12], this correlation can be taken into account by omitting  $u(p_r)$  in Eqn. (5 or 10) while calculating the standard uncertainty in  $\sigma_i$ . The corresponding value is denoted by  $u'$  which is also listed in Table 13. The weighted mean  $r$  of  $r_1$  and  $r_2$  is then calculated by

$$r = \frac{r_1 / u'^2(r_1) + r_2 / u'^2(r_2)}{1 / u'^2(r_1) + 1 / u'^2(r_2)} \quad (21)$$

The standard uncertainty of  $r$  at the target pressure  $P_t$  is

$$u(r) = \sqrt{\left(\frac{u(p_{rNPL})}{p_{rNPL}}\right)^2 r^2 + \left(\frac{u(p_{rNIST})}{p_{rNIST}}\right)^2 r^2 + \left(\frac{1}{u'^2(r_1)} + \frac{1}{u'^2(r_2)}\right)^{-1}} \quad (22)$$

The first two terms under the radical sign describe the influence of the uncertainty of the standard pressures  $p_{rj}$  that correlate to both  $r_1$  and  $r_2$  while the last term in the bracket describes all other influences e.g. due to rotor instability, offset determination, RD, temperature gradient and standard deviation of data. Ideally  $r$  should be equal to unity so that its departure from unity will give the relative difference between the two primary standards, i.e.,

$$d = r-1 \quad (23)$$

Also

$$u(d) = u(r). \quad (24)$$

Equivalence is assumed if

$$d \leq 2u(d) = U(d) \quad (25)$$

or

$$|E_{ni}| \leq 1 \quad (26)$$

with

$$E_n = \frac{d}{U(d)} \quad (27)$$

The results of calculations based on Eqs. (21) to (27) are given in Table 14.

Table 14. Values of  $r$ ,  $d$ ,  $U(d)$  and  $E_n$ .

$P_t$	$r$	$d$	$U(d)$	$E_n$
0.05 Pa	1.0013	0.0013	0.0053	0.2496

## 11. Discussion and conclusions

In the present comparison the degree of equivalence between the nominal pressure generated by the vacuum primary standards of NPL of India and NIST was tested. The values, of the calibration constants of the two SRGs used as transfer standards, measured at the two participating laboratories showed agreement well within their limits of uncertainty at  $k = 1$  level. Both standards were thus found to be equivalent to each other for the pressure of 0.05 Pa. The fact that the SRG rotors were hand carried without their thimbles for measurements at NIST implied that repeatable measurements are possible without the need to enclose the thimbles under vacuum using devices fitted to hold them in place during transit.

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