

**Bilateral Comparison of 100 pF Capacitance Standards**  
**(ongoing BIPM key comparison BIPM.EM-K14.b)**  
**between the NML, Ireland and the BIPM, June-October 2007**

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**Introduction**

This bilateral comparison between the NML (Ireland) and the BIPM was carried out from June to October 2007. Two 100 pF travelling standards belonging to the BIPM were used. The comparison was carried out with an 'A-B-A' pattern of measurements; the standards were measured first at the BIPM for a period of one month, then for a similar period at the NML, and finally again at the BIPM. The measurand was the two terminal-pair capacitance at a frequency of 1000 Hz and for a measuring voltage of 15 V. The BIPM was the pilot laboratory, and the comparison forms part of the ongoing BIPM key comparison BIPM.EM-K14.b.

**Travelling standards**

The two BIPM travelling standards are Andeen-Hagerling model AH11A capacitance modules having nominal values of 100 pF (S/N 01312 and S/N 01313) mounted in a frame model AH1100 (S/N 00077). The effect of ambient temperature on the standards mounted in the frame has been tested in the range 20 °C to 25 °C. No changes in the capacitance values greater than 1 part in 10<sup>8</sup> were detected over this range. Both laboratories measured the travelling standards at ambient temperatures between 22.5 °C and 23 °C. In these conditions, the temperature corrections are negligible. The effects of normal variations in atmospheric pressure and humidity are also negligible. The AH1100 frame was shipped between the two laboratories by standard air freight.

**Measurement principle**

*NML capacitance standard and measurement method*

The NML standard for capacitance comprises three commercially produced thermo-regulated fused silica capacitance standards ( 2 x Gen Rad 1408, 1 x AH 11A), each of nominal value 100 pF. The capacitance value ascribed to this group standard is the weighted mean of the individual drift-corrected values of the individual capacitors, which, in turn, are obtained from temporal drift models, based on previously certified values. Frequent internal comparisons between the individual standards are used to detect any anomalous drift behaviour.

The ratios of the BIPM travelling standards to the NML reference standards were measured using a substitution method. The transfer standard was a high resolution digital capacitance meter (Andeen Hagerling Type 2500A Capacitance Bridge). Two 1 metre long coaxial cables were used to connect the standards to the input of the capacitance bridge. Each bridge reading

was corrected for the effects of the inter-cable capacitance and the bridge offset. The nominal measuring voltage was 15 V, and the nominal frequency was 1 kHz.

### *BIPM capacitance standard and measurement method*

The BIPM maintains a reference group of four fused silica 10 pF capacitors (one of the NBS type and three of the GR 1408-A type). Since 1999, the mean value of the group has been measured twice a year using a measurement chain linking the 10 pF capacitances to the recommended value of the von Klitzing constant,  $R_{K-90} = 25\,812.807\ \Omega$ . The chain includes a capacitance bridge with ratio 10/1, a multi-frequency quadrature bridge, an ac-dc coaxial resistor with calculable frequency dependence of resistance, and a quantum Hall device operated at 1 Hz. The relative drift rate of the mean value of the reference group is about -3.5 parts in  $10^8$  per year.

The travelling standards were measured in terms of the BIPM 10 pF reference group using a coaxial bridge for two terminal-pairs capacitances with calibrated 10/1 ratio. The measurements were made using the standard BIPM conditions: nominal frequency of 1592 Hz and nominal voltage of 10 V. The frequency and voltage coefficients of the standards were separately measured against the known properties of the 10 pF reference. The change between 1592 Hz at 10 V and 1000 Hz at 15 V was found to be negligible.

## **Results**

### *Elimination of standard 01313 from the comparison*

*The results obtained with standard 01313 have not been used in this report. Due to a fault in the temperature control circuit of this standard during the comparison, the measurements give no useful information. The details of the problem, and the reasons for excluding the results are given in annex B. Only the results for standard 01312 are presented here in the main text.*

Table 1 gives the values assigned to the capacitance of standard 01312 on 31/08/2007 by BIPM and NML, along with the resulting relative difference,  $\Delta$ , and the corresponding uncertainties. All the uncertainties are given as relative standard uncertainties ( $1\sigma$  estimates). 31/08/2007 is the mean date of the NML measurements. Details of the uncertainty budgets are given in annex A.

The NML value is calculated as a simple mean of 18 measurements over the period 20/08/2007 to 12/09/2007. The NML uncertainty budget is dominated by the component for the reference standard, which is correlated between all measurements, so the uncertainty of the mean is the same as that for a single measurement.

A linear fit has been made to the BIPM ‘before’ and ‘after’ measurements, and the BIPM value is the value predicted by this fit for the date 31/08/2007. The uncertainty is composed of a global component for the BIPM traceability which is correlated between all measurements, and a type A component determined from the linear fit.

Figure 1 shows all the individual measurements of both BIPM and NML. The mean value of the NML measurements is shown, along with its uncertainty bar ( $1\sigma$ ). The linear fit to the BIPM before and after measurements is also shown, along with the  $1\sigma$  confidence bands of

the fitted line. From this plot it is clear that a linear fit is not the ideal model for the behaviour of the standard. The uncertainty contribution from the fit has been increased to reflect this, and since this is not a limiting contribution to the overall comparison uncertainty, a more complex treatment of the behaviour of the standard has not been sought.

The result can be summarised in the form of a degree of equivalence,  $D_{\text{NML}}$ , between NML and BIPM for measurements of 100 pF standards at 1 kHz and 15 V, with its associated expanded uncertainty,  $U_{\text{NML}}$  ( $k=2$ , 95% confidence):

$$D_{\text{NML}} = (C_{\text{NML}} - C_{\text{BIPM}}) / 100 \text{ pF} = + 0.01 \times 10^{-6}$$

$$U_{\text{NML}} = 0.78 \times 10^{-6}$$

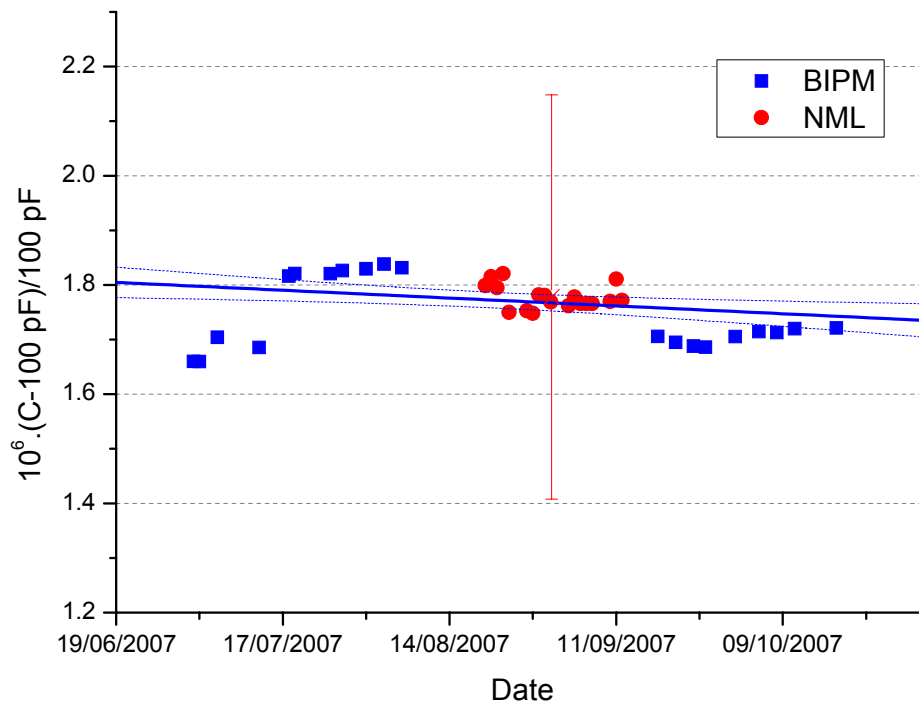
### Comments

The elimination of one of the two travelling standards from this comparison has not adversely affected the final result. The transport uncertainty for the remaining single standard does not dominate the final uncertainty.

The calibration and measurement capability (CMC) of NML held in the BIPM key comparison database appendix C for measurements of 100 pF standards at 1 kHz gives an expanded uncertainty ( $k=2$ , 95% confidence) of 200 aF ( $2 \times 10^{-6}$  in relative value). The result of this comparison is clearly consistent with this CMC.

**Table 1:** Final values for standard 01312

	NML	BIPM
$C_{01312}$ on 31/08/2007	100.000 178 pF	100.000 177 pF
Relative total uncertainty ( $1\sigma$ est)	$0.37 \times 10^{-6}$	$0.11 \times 10^{-6}$
$\Delta = (C_{\text{NML}} - C_{\text{BIPM}}) / C_{\text{BIPM}}$	$0.01 \times 10^{-6}$	
Combined uncertainty on $\Delta$ ( $1\sigma$ est)	$0.39 \times 10^{-6}$	



## Annex A: Uncertainty budgets

### *NML Uncertainty Budget*

The model of the measurement is :

$$C_X = r_X * (C_R + \delta_T) + \delta_C$$

where

$C_X$  is the value of the unknown capacitance;

$r_X$  is the ratio of the unknown capacitance to the NML reference standard, as evaluated from the capacitance bridge readings;

$C_R$  is the value assigned to the value of the NML reference standard on the date of the measurement (this is the weighted mean of the drift corrected values of the three individual reference capacitors that form the reference standard);

$\delta_T$  is the temperature correction to the NML standard;

$\delta_C$  is the correction due to uncorrected stray capacitance.

The corresponding uncertainty budget is presented in the table below:

Quantity	Estimate	Standard Uncertainty	Sensitivity Coefficient	Uncertainty Contribution
$r_X$	0.999 988 44	0.000 000 05	100 pF	5 aF
$C_R$	100.001 339 pF	35 aF	1	35 aF
$\delta_T$	0 aF	10 aF	1	10 aF
$\delta_C$	0 aF	5 aF	1	5 aF
$C_X$	100.001 83 pF			37 aF

The resultant standard uncertainty associated with a single reported value of a BIPM 100 pF travelling standard is 37 aF.

### *BIPM Uncertainty Budget*

Component	Relative uncertainty/ $10^{-9}$
Values at 1 Hz of 51.6 k $\Omega$ resistors used in quadrature bridge, with respect to $R_{K-90}$	14
1 Hz – 1541 Hz difference of 51.6 k $\Omega$ resistors	31
Operation of quadrature bridge at 1541 Hz	13
Scaling from 2000 pF capacitors of quadrature bridge to 10 pF reference	15
Link between 100 pF and 10 pF reference group	14
Extrapolation of the value of the 10 pF reference group	15
Evaluation of the change from 1592 Hz to 1000 Hz	30
Evaluation of the voltage coefficient from 10 V to 15 V	10
Type A uncertainty from linear fit	100
<b>Total</b>	<b>113</b>

All values are standard uncertainties ( $1\sigma$  estimates).

## **Annex B: Elimination of standard 01313**

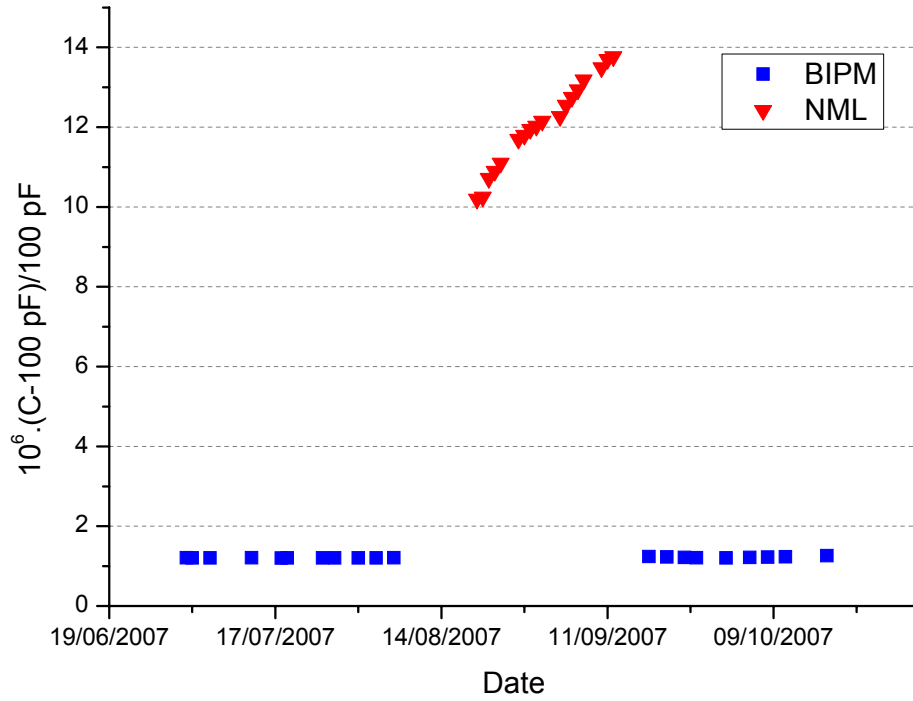
Two standards of the same nominal value were used in this comparison in order to try to reduce the influence of transport effects on the final result. The analysis would normally take a mean of the differences observed for the two standards, which would hopefully allow a reduction of the uncertainty due to the behaviour of the standards. Unfortunately in this case, the redundancy provided by the two standards was definitely needed, as standard 01313 developed a fault.

During the initial measurements at BIPM, there was no indication of a problem, but on arrival at NML, it was reported that the ‘Drift’ display on the chassis was out of range. (This display gives a reading derived from the difference between two temperature sensors within the capacitance unit; an out of range reading is an indication that there is some problem with the temperature control circuit.) The capacitance value observed at NML also proved to be very unstable (see figure B1). A visual check on the integrity of the internal connections to standard 01313 was carried out at NML but no fault was found. The power to the standards was not interrupted at any time during their stay at the NML.

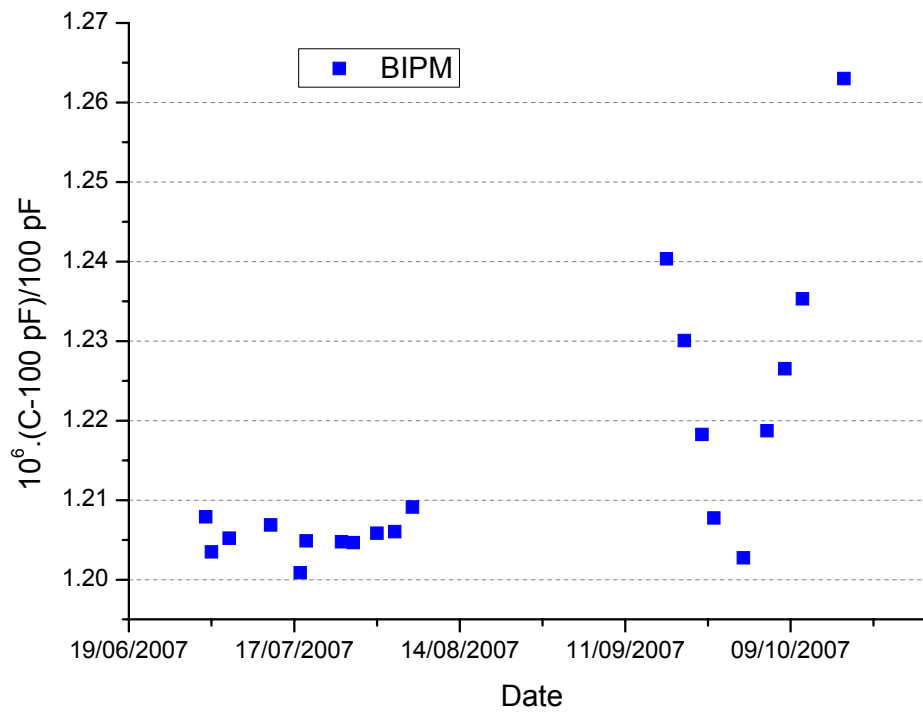
On the return to BIPM, the ‘Drift’ display was normal again, and the capacitance value was reasonably close to the value before transport. On the scale of plot B1, it appears that the problem only existed during the measurements at NML. Figure B2, however, presents only the BIPM before and after, measurements, and on this scale it is clear that the standard still had a problem after the return to BIPM. The most likely explanation is that there is an intermittent fault with the temperature control circuit.

The behaviour shown in figure B2 and the anomalous ‘Drift’ indication at NML give reason to eliminate this standard independently of the NML measurement results. The fact that the measurements of standard 01312 (same nominal value, measured using the same measurement system) were normal also supports the position that data of figure B1 are due to the behaviour of the standard, and not to any measurement problem at NML.

For the above reasons, it was decided to eliminate the measurements of standard 01313 from the calculation of the comparison result. The behaviour of the single remaining standard 01312 was sufficiently good that comparison was still a good test of NML’s traceability at the level of the stated uncertainties.



**Figure B1:** BIPM and NML measurements of standard 01313



**Figure B2:** as figure B1, but enlarged scale to show details of BIPM measurements