BUREAU INTERNATIONAL
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Bilateral Comparison of 1 Ω standards (ongoing BIPM key comparison BIPM.EM-K13.a) between the NIST (USA) and the BIPM.

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1 Introduction

A comparison of values assigned to 1 Ω resistance standards was carried out between the BIPM and the NIST (USA) in the period April 2007 to October 2007.

Three 1 Ω BIPM travelling standards of CSIRO type were calibrated first at the BIPM, then at the NIST and again at the BIPM after their return. The measurement periods are referred to as:

'Before' measurements at the BIPM: April and June-July 2007
NIST measurements: August-September 2007
'After' measurements at the BIPM: September-October 2007

The BIPM calibrations are corrected to the reference temperature 23.000 °C and the reference pressure 1013.25 hPa.
According to the protocol, the NIST does not apply pressure and temperature corrections to its results. The corrections are made by the BIPM, using the temperature and pressure coefficients of the standards together with the temperature and pressure measurements provided by the NIST.

There is no clear evidence of a single linear drift of each standard over the whole period of the comparison (three measurement periods, 'Before', 'NIST' and 'After': see Figures 1, 2 and 3). During each period, the resistance of each standard is therefore assumed to be constant, with superimposition of a random noise.

For each period, the calibration value assigned to each standard is the mean value of the measurements performed during this period, with an associated standard uncertainty.

The difference between the NIST and the BIPM calibrations of a given standard R_i can be written as:

$$\Delta_i = R_{\mathrm{NIST},\,i} - R_{\mathrm{BIPM},\,i}$$

If three standards are used, the mean of the differences is:

$$\Delta_{\mathrm{NIST-BIPM}} = \frac{1}{3}\sum_{i=1}^{3}(R_{\mathrm{NIST},\,i} - R_{\mathrm{BIPM},\,i}) \qquad (1)$$

This expression can also be written as:

$$\Delta_{\mathrm{NIST-BIPM}} = \frac{1}{3}\sum_{i=1}^{3}R_{\mathrm{NIST},\,i} - \frac{1}{3}\sum_{i=1}^{3}R_{\mathrm{BIPM},\,i} \qquad (2)$$

which is the difference of the means.

2 Measurements at the BIPM

2.1 BIPM calibrations

The BIPM measurements were carried out by comparison with a 100 Ω reference resistor (referred to as BI100-3) whose value is known with respect to the BIPM quantized Hall resistance (QHR) standard. The comparison is performed using a DC cryogenic current comparator operating with a 50 mA current in the 1 Ω resistors.
In order to minimize the interpolation uncertainty, the 100 Ω reference resistor was calibrated four times against the QHR in the period April 2007 to October 2007.

The 1 Ω resistors were kept in a temperature controlled oil bath at a temperature which is close (within a few mK) to the reference temperature. The temperature of the standards is determined by means of a calibrated SPRT, in conjunction with thermocouples.

During the first period of measurements at the BIPM, the medium-term dispersion was larger than usual (standard deviation in the range 5 to 7 parts in 10^{-9} instead of 3 to 4 parts in 10^{-9}). This could be attributed to the perturbations produced by the refurbishment works in other laboratory rooms in the same building.

The BIPM measurements are summarized in Table 1 and the uncertainty budget in Table 2.

BIPM	**Relative difference from nominal 1 Ω value / 10^{-6}**			
Standard #	BEFORE	Std. dev. u_{1B}	AFTER	Std. dev. u_{1A}
64193	– 5.165	0.0073	– 5.188	0.0040
64200	– 0.762	0.0054	– 0.795	0.0033
64207	– 0.538	0.0064	– 0.538	0.0044
Mean value of 'Before' and 'After'				
Standard #	**mean** / 10^{-6}	Dispersion u_1 / 10^{-9}	Systematic u_2 / 10^{-9}	
64193	**– 5.177**	4.2	16	
64200	**– 0.778**	3.2	16	
64207	**– 0.538**	3.9	16	

Table 1: Summary of the BIPM calibrations. The dispersion is estimated by the standard deviations, and 'systematic' refers to the sources of uncertainty that do not contribute to the variability of the results.

Source of uncertainty	relative standard uncertainty
Imperfect realisation of R_{K-90}	2 x 10^{-9}
Calibration of the BIPM 100 Ω reference (BI100-3) against R_{K-90}	3 x 10^{-9}
Interpolation / extrapolation of the value of BI100-3	13 x 10^{-9}
Measurement of the (1 Ω / BI100-3) ratio	8 x 10^{-9}
Temperature correction for the 1 Ω standard	2 x 10^{-9}
Pressure correction for the 1 Ω standard	3 x 10^{-9}
Combined uncertainty	**16 x 10^{-9}**

Table 2: BIPM uncertainty budget for the calibration of the 1 Ω travelling standards.

The value attributed to *i*-th standard is the arithmetic mean of the "Before" and "After" values.

$$R_{\text{BIPM, i}} = (R_{\text{Before}, i} + R_{\text{After, i}})/2$$

For each standard, the uncertainty u_1 associated with the dispersion is the quadratic mean of the standard deviations "Before" and "After".

$$u_{1,i}^2 = (u_{1\text{Before},i}^2 + u_{1\text{After},i}^2) / 2^2$$

u_2 is the uncertainty arising from the combined contributions associated with the BIPM measurement facility and the traceability, as described in Table 2. This component is assumed to be strongly correlated between calibrations performed in the same period.

For a single standard, the BIPM uncertainty $u_{\text{BIPM},\,i}$ is obtained from: $u_{\text{BIPM},\,i}^2 = u_{1,\,i}^2 + u_{2,\,i}^2$

The $u_{2,\,i}$ are assumed to be correlated, unlike $u_{1,\,i}$.

Using expression (2), when the mean (for three standards) of the NIST-BIPM relative difference is calculated, the BIPM contribution to the uncertainty is:

$$u_{\text{BIPM}}^2 = \sum_{i=1}^{3} \frac{u_{1,\,i}^2}{3^2} + u_2^2 \qquad (3)$$

Using the values shown in Table 2, the relative standard uncertainty u_{BIPM} is

$$u_{\text{BIPM}} = 16.1 \times 10^{-9}.$$

2.2 Uncertainty associated with the transfer

u_{d} is the uncertainty associated with the drift (or the step changes) of the travelling standards observed after their return at the BIPM.
The final resistance value attributed by the BIPM (the mean of the 'Before' and 'After' measurements) is in the middle of the step d: $d = \left|(R_{\text{After}} - R_{\text{Before}})\right|$

As we have no clear knowledge about the behaviour of the standards during the period between 'Before' and 'After', it is assumed that the actual resistance could have had any value in the range d, with equal probability.

Assuming a rectangular probability distribution, $u_{\text{d}} = \frac{d}{2} \cdot \frac{1}{\sqrt{3}}$

Another source of uncertainty associated with the transfer is the difference in the operating currents used by the two laboratories: 50 mA at the BIPM and 100 mA at the NIST. This might influence the resistance of the standards through their power coefficients.
A series of measurements were performed at the BIPM to determine these power coefficients. Unfortunately, our current source does not allow currents as large as 100 mA, and similarly the NIST facility can not easily be modified to operate at 50 mA.
The BIPM series of measurements were therefore carried out using alternately I = 50 mA and $I/\sqrt{2}$, but no significant change could be observed within the noise of the measurements, that is about 1 part in 10^9.

A conservative value of the relative standard uncertainty u_{P} associated with possible power effects in the range 50 mA – 100 mA was estimated to be:

$$u_{\text{P}} = 2 \times 10^{-9}.$$

For a single standard, the transfer uncertainty $u_{\text{T},\,i}$ is obtained from: $u_{\text{T},\,i}^2 = u_{\text{d},\,i}^2 + u_{\text{P},\,i}^2$

The $u_{\text{P},\,i}$ are assumed to be correlated, unlike $u_{\text{d},\,i}$.

Following the same reasoning as in expression (3), the uncertainty u_T associated with the transfer (for the mean of three standards) is:

$$u_T^2 = \sum_{i=1}^{3} \frac{u_{d,i}^2}{3^2} + u_P^2$$

	TRANSFER	
Standard #	Drift u_d / 10^{-9}	Power u_P / 10^{-9}
64193	6.7	2
64200	9.6	2
64207	0.1	2
Combined	3.9	2
Total u_T	**4.4**	

Table 3: Uncertainty associated with the drift and the power coefficient of the standards.

Using the values of Table 3, the relative standard uncertainty u_T is:

$$u_T = 4.4 \times 10^{-9}$$

3 NIST results

Serial No. of item(s): **64193, 64200, 64207** Manufacturer: CSIRO	Standard uncertainty	Distribution /method of evaluation	Sensitivity coefficient	Uncertainty contribution	Degrees of freedom
Influence factor y_i	$u(y_i)$	*Method/(A, B)*	c_i	$u(R_i)$ (Ω)	ν_i
Scaling / traceability	1.00E-08	Normal/B	1 Ω	1.00E-08	100
Reference standards	6.50E-09	Normal/B	1 Ω	6.50E-09	∞
Measuring apparatus	4.00E-09	Rectangular/B	1 Ω	4.00E-09	∞
Ambient conditions:					
Temperature	2.00E-03 °C	Normal/B			∞
Pressure	1.00E-01 kPa	Normal/B			∞
Standard deviation	2.90E-09	Normal/A	1 Ω	2.90E-09	31

Table 4: NIST uncertainty budget (typical values; the standard deviation value is given as an example).

Method: The NIST quantum Hall effect standard was compared twice each year to five 100 Ω standards using a cryogenic current comparator bridge. Four times each year, these five standards were compared to two or three 1 Ω standards of the CSIRO design, using a second cryogenic current comparator. These CSIRO-type standards were then used as transfer standards to maintain the mean reference value of a group of five Thomas-type 1 Ω standards in an automated potentiometer measurement system based on a commercial current comparator bridge (Guildline 9930). The 1 Ω comparison standards were compared to this reference group using the same potentiometric system. Measurements were repeated on 32 days in the NIST segment of the comparison. All 32 measurements were used to determine the average resistance value relative to the NIST measurement standards.

Measurement temperature control: The reference standards were maintained in a mineral oil bath maintained at 25.000 °C as measured by a calibrated SPRT and thermometer bridge (Hart 5175). The BIPM comparison standards were measured in a mineral oil bath maintained at nominal 23.00 °C as measured by a calibrated thermistor probe (Fluke 5640). The thermistor measurements were corrected using a calibrated SPRT and thermometer bridge (Hart 5175).
Test current: Direct current with reversal, measured at 100 mA.
Humidity: Relative humidity in the laboratory averaged 40%.

Serial No. of item:	Average measurement date, time	Resistance value (Ω)	Standard deviation value (Ω)	Temperature	Barometric pressure including mineral oil
64193	25-Aug-2007, 01:00	0.999 994 818	3.6×10^{-9}	22.9984 °C	101.05 kPa
64200	25-Aug-2007, 01:00	0.999 999 220	2.9×10^{-9}	22.9984 °C	101.05 kPa
64207	25-Aug-2007, 01:00	0.999 999 430	2.8×10^{-9}	22.9984 °C	101.05 kPa

Table 5: Summary of the NIST calibrations.

The NIST results are corrected to the reference temperature and the reference pressure using the coefficients shown in Table 6.

	Relative temperature coefficients		Relative pressure coefficients.
Standard #	Alpha $_{23}$ / (10^{-6}/K)	Beta / (10^{-6}/K²)	/ (10^{-9}/hPa)
64193	– 0.004	– 0.001	– 0.17
64200	– 0.007	0.000	– 0.23
64207	– 0.009	0.000	– 0.25

Table 6: Temperature and pressure coefficients of the travelling standards.

Reference temperature = 23.000°C Reference pressure = 1013.25 hPa		
	Relative corrections applied to the NIST results	
Standard #	For temperature	For pressure
64193	$- 6.4 \times 10^{-12}$	$- 4.7 \times 10^{-10}$
64200	$- 1.1 \times 10^{-11}$	$- 6.3 \times 10^{-10}$
64207	$- 1.4 \times 10^{-11}$	$- 6.9 \times 10^{-10}$

Table 7: Corrections for temperature and pressure applied to the NIST results.

There is no additional uncertainty associated with these corrections. The contributions arising from the uncertainties associated with the temperature and pressure measurements at NIST are negligible due to the very small size of the corrections.

In Table 8, u_1 is the uncertainty associated with the dispersion (see Table 5, column 4) and u_2 combines the other contributions (first three lines in Table 4).

NIST After corrections	Resistance / Ω	Relative difference from nominal. / 10^{-6}	Relative standard uncertainties	
			Dispersion u_1 / 10^{-9}	Others u_2 / 10^{-9}
64193	0.999994818	– 5.1825	3.6	12.6
64200	0.999999219	– 0.7806	2.9	12.6
64207	0.999999429	– 0.5707	2.8	12.6

Table 8: Summary of the NIST results, after corrections for temperature and pressure.

For a single standard, the NIST uncertainty $u_{\mathrm{NIST},\,i}$ is obtained from: $u^2_{NIST,\,i} = u^2_{1,\,i} + u^2_{2,\,i}$

The $u_{2,\,i}$ are assumed to be correlated, unlike the $u_{1,\,i}$.

Using expression (2), when the mean (for three standards) of the NIST-BIPM relative difference is calculated, the NIST contribution to the uncertainty is:

$$u^2_{\mathrm{NIST}} = \sum_{i=1}^{3} \frac{u^2_{1,\,i}}{3^2} + u^2_2 \qquad (5)$$

Using the values shown in Table 8, the relative standard uncertainty u_{NIST} is

$$u_{\mathrm{NIST}} = 12.7 \times 10^{-9}.$$

4 Comparison NIST – BIPM

The differences between the values assigned by the NIST at the NIST, R_{NIST}, and those assigned by the BIPM at the BIPM, R_{BIPM}, to each of the three travelling standards during the period of the comparison are shown in Table 9.

NIST - BIPM	
Standard #	$(R_{NIST} - R_{BIPM}) / (1\ \Omega) / 10^{-6}$
64193	– 0.006
64200	– 0.002
64207	– 0.033
mean	**– 0.014**

Table 9: Differences between the values assigned by the NIST (R_{NIST}) and by the BIPM (R_{BIPM}) to the three travelling standards.

The mean difference between the NIST and the BIPM calibrations is:

$$(R_{NIST} - R_{BIPM}) / (1\ \Omega) = -0.014 \times 10^{-6}$$

The relative combined standard uncertainty of the comparison, u_C, is:

$$u_C^2 = u_{BIPM}^2 + u_{NIST}^2 + u_T^2$$

where $u_{BIPM} = 16.1 \times 10^{-9}$,
$u_{NIST} = 12.7 \times 10^{-9}$,
$u_T = 4.4 \times 10^{-9}$
as calculated in sections 2 and 3: $u_C = 0.021 \times 10^{-6}$

The NIST and the BIPM calibrations are in good agreement, with a difference smaller than one standard uncertainty.

The final result of the comparison is presented as the degree of equivalence D between the NIST and the BIPM for values assigned to 1 Ω resistance standards, and its expanded relative uncertainty (expansion factor $k = 2$) , U_C

$$D = [(R_{NIST} - R_{BIPM}) / 1\ \Omega] = -0.014 \times 10^{-6}$$

$$U_C = 0.042 \times 10^{-6}$$

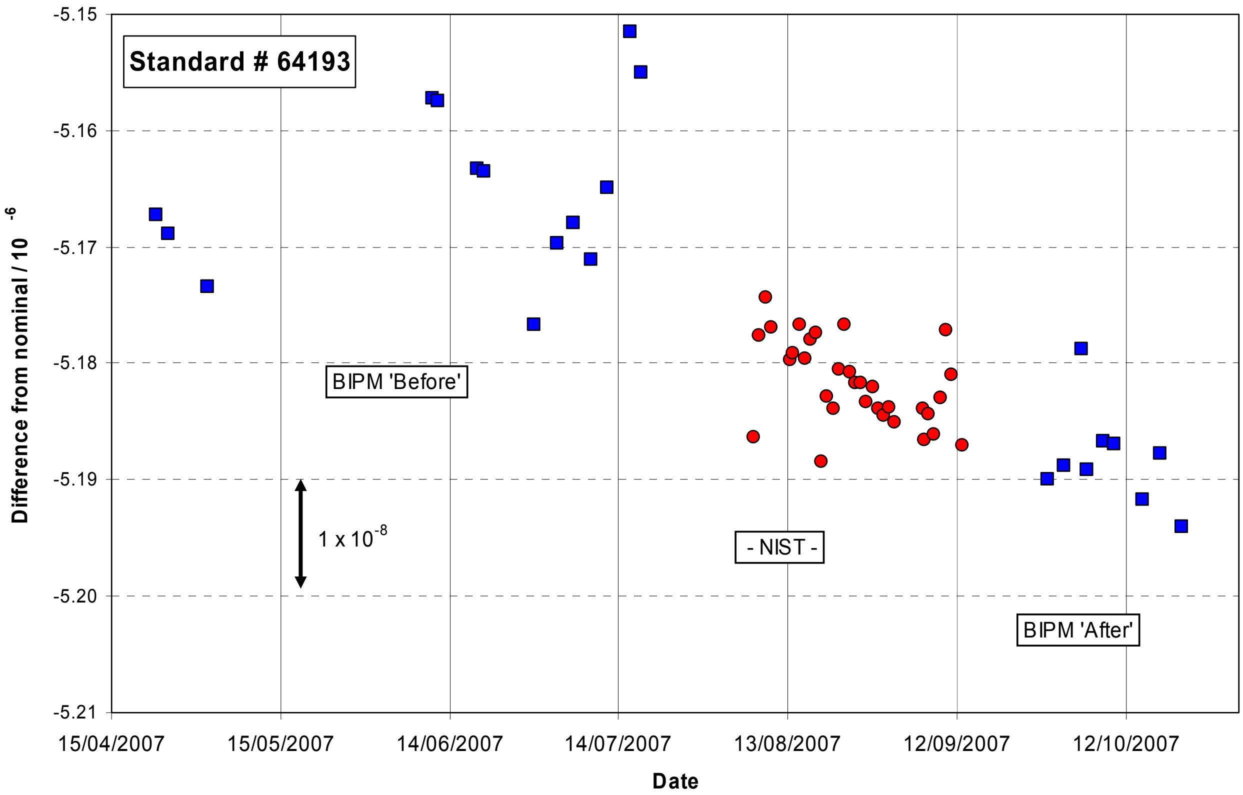

Figure 1: Calibrations at the BIPM (squares) and at the NIST (circles) of the travelling standard ref. 64193, expressed as the relative deviation from the nominal 1 Ω value.

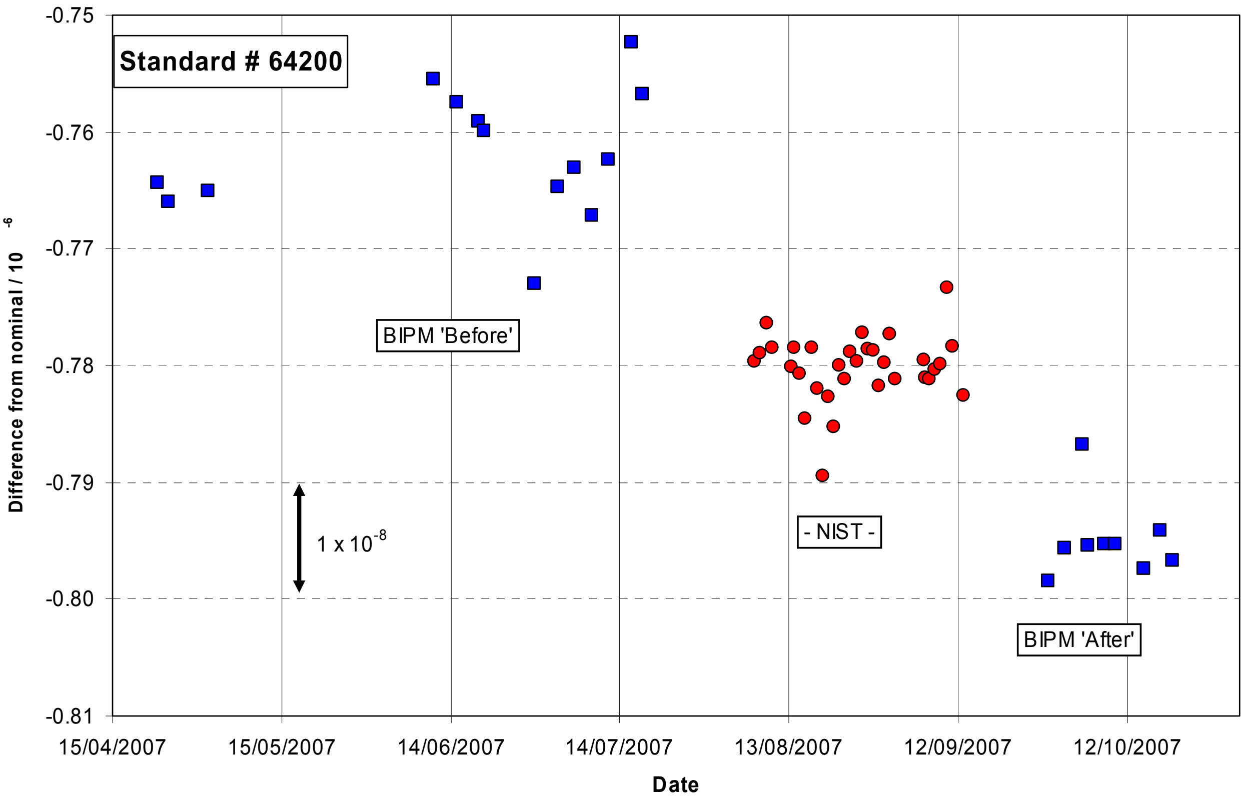

Figure 2: Calibrations at the BIPM (squares) and at the NIST (circles) of the travelling standard ref. 64200, expressed as the relative deviation from the nominal 1 Ω value.

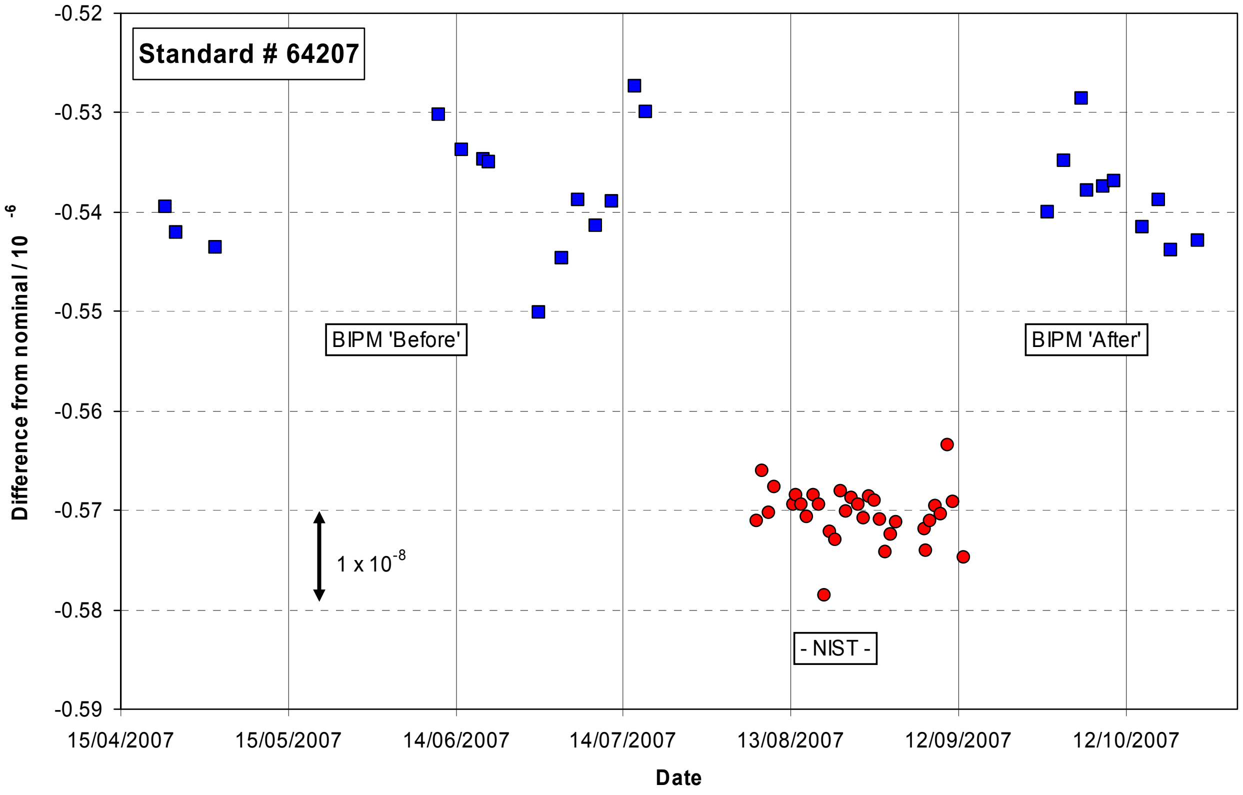

Figure 3: Calibrations at the BIPM (squares) and at the NIST (circles) of the travelling standard ref. 64207, expressed as the relative deviation from the nominal 1 Ω value.