BUREAU INTERNATIONAL DES POIDS ET MESURES

Bilateral Comparison of 1 Ω standards (ongoing BIPM key comparison BIPM.EM-K13.a) between the CMI (Czech Republic) and the BIPM

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Final report

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1 Introduction

A comparison of values assigned to 1 Ω resistance standards was carried out between the BIPM and the CMI (Czech Republic) in the period September 2007 to January 2008.

Two 1 Ω BIPM travelling standards of CSIRO type were calibrated first at the BIPM, then at the CMI and again at the BIPM after their return. The measurement periods are referred to as:
'Before' measurements at the BIPM: September-October 2007
CMI measurements: November-December 2007
'After' measurements at the BIPM: December 2007-January 2008

The BIPM calibrations are corrected to the reference temperature 23.000 °C and the reference pressure 1013.25 hPa.
According to the protocol, the CMI did not apply pressure and temperature corrections to its results. The corrections were made by the BIPM, using the temperature and pressure coefficients of the standards together with the temperature and pressure measurements provided by the CMI.
The calibration reports provided by the CMI are summarized by the BIPM in section 3 of the present report.

There is no evidence of a single linear drift of each standard over the whole period of the comparison (three measurement periods, 'Before', 'CMI' and 'After': see Figures 1 and 2). During each period, the resistance of each standard is therefore assumed to be constant, with superimposition of a random noise.

For each period, the calibration value assigned to each standard is the mean value of the measurements performed during this period, with an associated standard uncertainty.

The difference between the CMI and the BIPM calibrations of a given standard R_i can be written as:

$$\Delta_i = R_{\mathrm{CMI},\,i} - R_{\mathrm{BIPM},\,i}$$

If two standards are used, the mean of the differences is:

$$\Delta_{\mathrm{CMI-BIPM}} = \frac{1}{2}\sum_{i=1}^{2}(R_{\mathrm{CMI},\,i} - R_{\mathrm{BIPM},\,i}) \qquad (1)$$

This expression can also be written as:

$$\Delta_{\mathrm{CMI-BIPM}} = \frac{1}{2}\sum_{i=1}^{2}R_{\mathrm{CMI},\,i} - \frac{1}{2}\sum_{i=1}^{2}R_{\mathrm{BIPM},\,i} \qquad (2)$$

which is the difference of the means.

2 Measurements at the BIPM

2.1 BIPM calibrations

The BIPM measurements were carried out by comparison with a 100 Ω reference resistor (referred to as BI100-3) whose value is known with respect to the BIPM quantized Hall resistance (QHR) standard. The comparison was performed using a DC cryogenic current comparator operating with a 50 mA current in the 1 Ω resistors.
In order to minimize the interpolation uncertainty, the 100 Ω reference resistor was calibrated three times against the QHR in the period September 2007 to January 2008.
The 1 Ω resistors were kept in a temperature controlled oil bath at a temperature which is close (within a few mK) to the reference temperature. The temperature of the standards was determined by means of a calibrated SPRT, in conjunction with thermocouples.

The BIPM measurements are summarized in Table 1 and the uncertainty budget in Table 2.

BIPM	Relative difference from nominal 1 Ω value / 10^{-6}			
Standard #	BEFORE	Std. dev. u_{1B}	AFTER	Std. dev. u_{1A}
64193	– 5.188	0.0042	– 5.186	0.0062
64207	– 0.538	0.0044	– 0.567	0.0078
Mean value of 'Before' and 'After'				
Standard #	**mean** / 10^{-6}	Exp. Std. dev. u_1 / 10^{-9}	Systematic u_2 / 10^{-9}	
64193	**– 5.187**	3.7	16	
64207	**– 0.552**	4.5	16	

Table 1: Summary of the BIPM calibrations. The dispersion is estimated by the standard deviations, and 'systematic' refers to the sources of uncertainty that do not contribute to the variability of the results.

Source of uncertainty	relative standard uncertainty
Imperfect realisation of R_{K-90}	2 x 10^{-9}
Calibration of the BIPM 100 Ω reference (BI100-3) against R_{K-90}	3 x 10^{-9}
Interpolation / extrapolation of the value of BI100-3	13 x 10^{-9}
Measurement of the (1 Ω / BI100-3) ratio	8 x 10^{-9}
Temperature correction for the 1 Ω standard	2 x 10^{-9}
Pressure correction for the 1 Ω standard	3 x 10^{-9}
Combined uncertainty	**16 x 10^{-9}**

Table 2: BIPM uncertainty budget for the calibration of the 1 Ω travelling standards.

The value attributed to *i*-th standard is the arithmetic mean of the "Before" and "After" values.

$$R_{\text{BIPM, i}} = (R_{\text{Before}, i} + R_{\text{After, i}})/2$$

For each standard, the uncertainty u_1 associated with the dispersion is the quadratic mean of the standard deviations "Before" and "After".

$$u^2_{1,i} = (u^2_{1\text{Before}, i} + u^2_{1\text{After}, i})/2^2$$

u_2 is the uncertainty arising from the combined contributions associated with the BIPM measurement facility and the traceability, as described in Table 2. This component is assumed to be strongly correlated between calibrations performed in the same period.

For a single standard, the BIPM uncertainty $u_{\text{BIPM}, i}$ is obtained from: $u^2_{\text{BIPM}, i} = u^2_{1,i} + u^2_{2,i}$

The $u_{2,i}$ are assumed to be correlated, unlike $u_{1,i}$.

Using expression (2), when the mean (for two standards) of the CMI-BIPM relative difference is calculated, the BIPM contribution to the uncertainty is:

$$u_{\mathrm{BIPM}}^2 = \sum_{i=1}^{2} \frac{u_{1,i}^2}{2^2} + u_2^2 \qquad (3)$$

Using the values shown in Table 1, the relative standard uncertainty u_{BIPM} is

$$u_{\mathrm{BIPM}} = 16.3 \times 10^{-9}.$$

2.2 Uncertainty associated with the transfer

u_{d} is the uncertainty associated with the drift (or the step changes) of the travelling standards observed after their return at the BIPM.
The final resistance value attributed by the BIPM (the mean of the 'Before' and 'After' measurements) is in the middle of the step d: $d = |(R_{\mathrm{After}} - R_{\mathrm{Before}})|$

As we have no clear knowledge about the behaviour of the standards during the period between 'Before' and 'After', it is assumed that the actual resistance could have had any value in the range d, with equal probability.

Assuming a rectangular probability distribution, $u_{\mathrm{d}} = \frac{d}{2} \cdot \frac{1}{\sqrt{3}}$

Another source of uncertainty associated with the transfer can be the difference in the operating currents used by the two laboratories, influencing the resistance of the standards through their power coefficients. In the present case, the nominal operating currents are identical (50 mA) in the two laboratories.
The standards being used with the same current under similar conditions (oil-bath at 23 °C), the influence of their power coefficient is supposed to be similar in both laboratories. It is therefore assumed that the contribution from this common parameter is negligible when calculating the difference between the BIPM and the CMI calibrations. The value of the relative standard uncertainty u_{P} associated with possible power effects is estimated to be negligible.

For a single standard, the transfer uncertainty $u_{\mathrm{T},\,i}$ is obtained from: $u_{\mathrm{T},\,i}^2 = u_{\mathrm{d},\,i}^2 + u_{\mathrm{P},\,i}^2$
The $u_{\mathrm{P},\,i}$ are assumed to be correlated, unlike $u_{\mathrm{d},\,i}$.
Following the same reasoning as in expression (3), the uncertainty u_{T} associated with the transfer (for the mean of two standards) is:

$$u_{\mathrm{T}}^2 = \sum_{i=1}^{2} \frac{u_{\mathrm{d},\,i}^2}{2^2} + u_{\mathrm{P}}^2$$

	Transfer	
Standard #	Drift u_{d} / 10^{-9}	Power u_{P} / 10^{-9}
64193	0.7	0.0
64207	8.2	0.0
Combined	4.1	0.0
Total u_{T}	**4.1**	

Table 3: Uncertainty associated with the drift and the power coefficient of the standards.

Using the values of Table 3, the relative standard uncertainty u_{T} is:

$$u_{\mathrm{T}} = 4.1 \times 10^{-9}$$

3 Measurements at the CMI

3.1 Method of calibration:

The travelling standards were calibrated by direct comparison with a 100 Ω reference standard using a cryogenic current comparator (CCC), repeatedly, in a four-terminal configuration.
The 100 Ω reference standard is itself known in terms of the recommended value of the von Klitzing constant, R_{K-90} = 25 812.807 Ω, by calibration against a quantized Hall resistance.

3.2 Operating conditions:

The travelling standards were placed in an oil-bath thermoregulated at 23.00 °C ± 0.02 °C.
The actual temperature of the standards and the barometric pressure (including mineral oil) were recorded during each individual calibration.
Operating current: 50 mA dc.
Barometric pressure range: 969 hPa – 1002 hPa.

3.3 CMI results:

The standards were measured 12 times in the period 16 November – 14 December 2007.
The results are summarized in Table 4.

Serial No. of standard:	Resistance value (Ω)	Rel. standard deviation / 10^{-9}	Mean temperature	Mean barometric pressure, including mineral oil
64193	0.999 994 847	4	22.998 °C	985 hPa
64207	0.999 999 506	3	22.998 °C	985 hPa

Table 4: Summary of the CMI calibrations.

The CMI results are corrected to the reference temperature and the reference pressure using the coefficients shown in Table 5. The corrections, calculated by the BIPM, are shown in Table 6.

	Relative temperature coefficients		Relative pressure coefficients.
Standard #	Alpha$_{23}$ / (10^{-6}/K)	Beta / (10^{-6}/K²)	/ (10^{-9}/hPa)
64193	– 0.004	– 0.001	– 0.17
64207	– 0.009	0.000	– 0.25

Table 5: Temperature and pressure coefficients of the travelling standards.

Reference temperature = 23.000°C Reference pressure = 1013.25 hPa		
	Relative corrections applied to the CMI results	
Standard #	For temperature	For pressure
64193	– 8.0 x 10^{-12}	– 4.8 x 10^{-9}
64207	– 1.8 x 10^{-11}	– 7.1 x 10^{-9}

Table 6: Corrections for temperature and pressure applied to the CMI results.

The uncertainties u_{Temp} and u_{Press} associated with the temperature and pressure corrections applied by the BIPM are estimated to be $u_{\mathrm{Temp}} = 2 \times 10^{-9}$ and $u_{\mathrm{Press}} = 3 \times 10^{-9}$, leading to a combined uncertainty $u_3 = 3.6 \times 10^{-9}$.

Source of uncertainty	Relative standard uncertainty / 10^{-9}
Reference	
Calibration of the 100 Ω reference standard Rs	15
Drift extrapolation of the reference standard Rs	11.6
Measurement system	
Detection circuit	5.8
CCC turns ratio accuracy	11.6
Non-linearity	5.8
Others	
Temperature deviation of environment (air) on Rs	2.9
Power effect on Rs	5.8
Atmospheric pressure on Rs	5.8
Leakage currents	2.9
Connection	5.8
Repeatability (see Table 4)	
Combined (sum in quadrature): u_2	**26.1**

Table 7: Summary of the CMI uncertainty budget

CMI After corrections	Resistance / Ω	Relative difference from nominal. / 10^{-6}	Relative standard uncertainties		
			Dispersion u_1 / 10^{-9}	Systematic u_2 / 10^{-9}	Corrections u_3 / 10^{-9}
64193	0.999 994 842	– 5.1578	4	26.1	3.6
64207	0.999 999 499	– 0.5011	3	26.1	3.6

Table 8: Summary of the CMI results, after corrections for temperature and pressure.

For a single standard, the CMI uncertainty $u_{\mathrm{CMI},\,i}$ is obtained from: $u_{CMI,\,i}^2 = u_{1,\,i}^2 + u_{2,\,i}^2 + u_{3,\,i}^2$

The $u_{2,\,i}$ and $u_{3,\,i}$ are assumed to be correlated, unlike the $u_{1,\,i}$.

Using expression (2), when the mean (for two standards) of the CMI-BIPM relative difference is calculated, the CMI contribution to the uncertainty is:

$$u_{\mathrm{CMI}}^2 = \sum_{i=1}^{2} \frac{u_{1,\,i}^2}{2^2} + u_2^2 + u_3^2 \qquad (5)$$

Using the values shown in Table 8, the relative standard uncertainty u_{CMI} is

$$u_{\mathrm{CMI}} = 26.5 \times 10^{-9}.$$

4 Comparison CMI – BIPM

The differences between the values assigned by the CMI at the CMI, R_{CMI}, and those assigned by the BIPM at the BIPM, R_{BIPM}, to each of the two travelling standards during the period of the comparison are shown in Table 9.

CMI - BIPM	
Standard #	$(R_{\mathrm{CMI}} - R_{\mathrm{BIPM}})$ / (1 Ω) / 10^{-6}
64193	0.029
64207	0.051
mean	**0.040**

Table 9: Differences between the values assigned by the CMI (R_{CMI}) and by the BIPM (R_{BIPM}) to the two travelling standards.

The mean difference between the CMI and the BIPM calibrations is:

$$(R_{\mathrm{CMI}} - R_{\mathrm{BIPM}}) / (1\ \Omega) = 0.040 \times 10^{-6}$$

The relative combined standard uncertainty of the comparison, u_{C}, is:

$$u_{\mathrm{C}}^2 = u_{\mathrm{BIPM}}^2 + u_{\mathrm{CMI}}^2 + u_{\mathrm{T}}^2$$

where $u_{\mathrm{BIPM}} = 16.3 \times 10^{-9}$,
$u_{\mathrm{CMI}} = 26.5 \times 10^{-9}$,
$u_{\mathrm{T}} = 4.1 \times 10^{-9}$
as calculated in sections 2 and 3: $\boldsymbol{u_{\mathrm{C}} = 0.031 \times 10^{-6}}$

The final result of the comparison is presented as the degree of equivalence D between the CMI and the BIPM for values assigned to 1 Ω resistance standards, and its expanded relative uncertainty (expansion factor $k = 2$, corresponding to a confidence level of 95 %) , U_{C}

$$D = [(R_{\mathrm{CMI}} - R_{\mathrm{BIPM}}) / 1\ \Omega] = 0.040 \times 10^{-6}$$

$$U_{\mathrm{C}} = 0.062 \times 10^{-6}$$

The CMI and the BIPM calibrations are in good agreement, with a difference smaller than the expanded uncertainty.

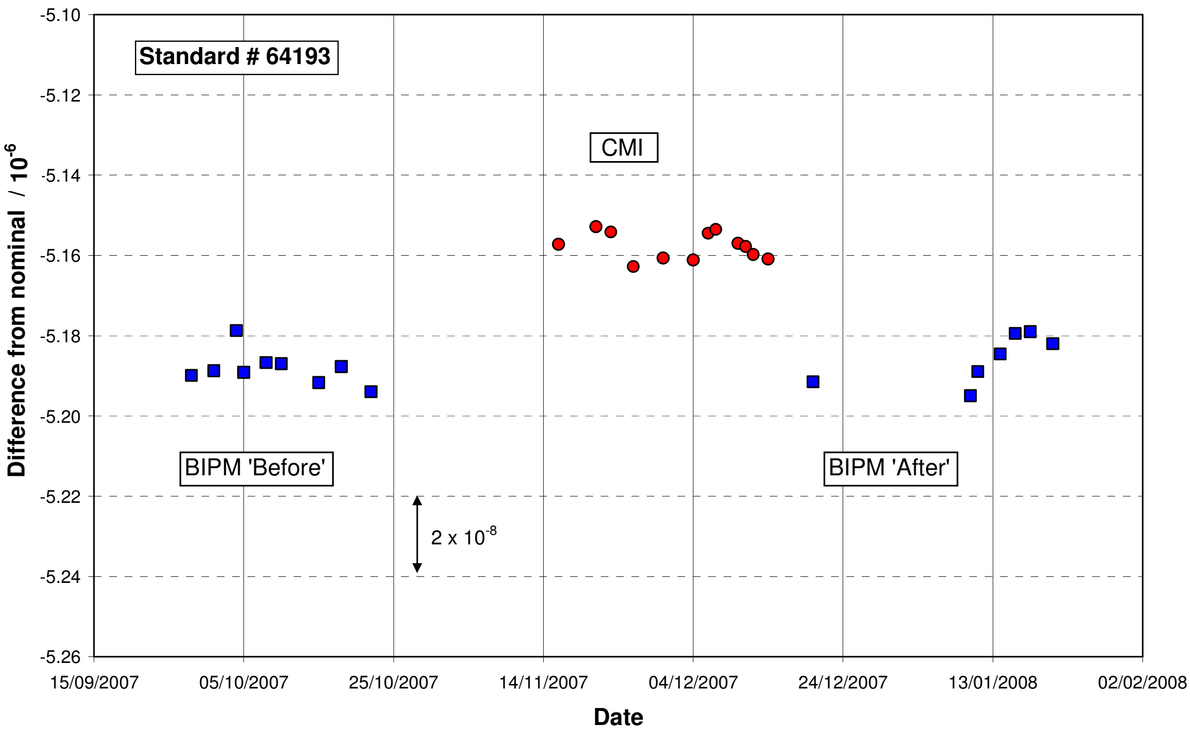

Figure 1: Calibrations at the BIPM (squares) and at the CMI (circles) of the travelling standard ref. 64193, expressed as the relative deviation from the nominal 1 Ω value.

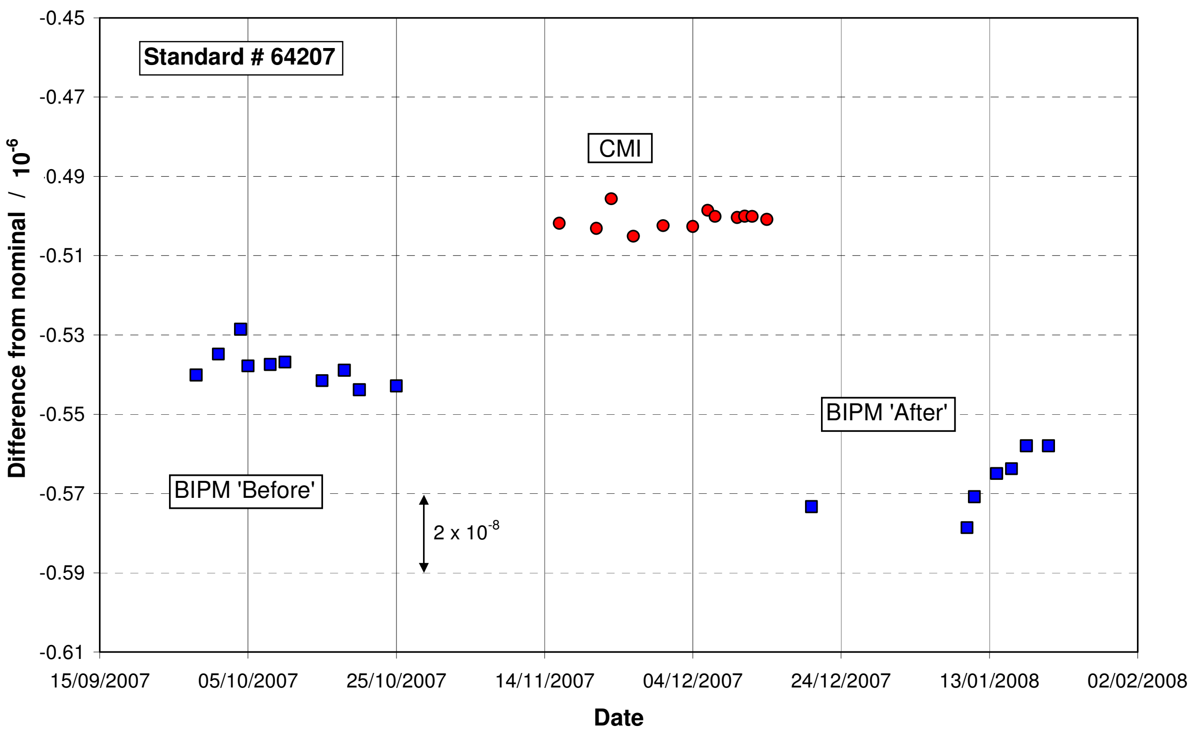

Figure 2: Calibrations at the BIPM (squares) and at the CMI (circles) of the travelling standard ref. 64207, expressed as the relative deviation from the nominal 1 Ω value.